A DCC-GARCH MODEL TO ESTIMATE THE RISK TO THE CAPITAL MARKET IN ROMANIA

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Abstract

In this paper we propose to study if the standard and asymmetric dynamic conditional correlation (DCC) models, following Cappiello et al. (2006), may capture spillover effects and the degree of interaction with the European capital market using the DAX index as proxy.

We found evidence that the asymmetric DCC models perform better than the similar non-asymmetric ones. In the second semester of 2011, increased significant dynamic correlations suggest the presence of volatility spillovers from the main capital equity markets. Although all DCC models can capture contagion, seen as a significant increase in the co-movements of stock index returns, the AGD-DCC model is more sensitive to unexpected changes in returns. The results indicate significant, but not very strong correlation of BET and BETFI indexes with the DAX index in the second semester of 2011.

Keywords: volatility spillovers, contagion effects, stock return comovements

JEL Classification: G01, G14, G32

I. Introduction

While the univariate GARCH models can analyze the variance of the market shock in the univariate asymmetric models, the recent development of the class of multivariate GARCH models led to incorporation of the asymmetric response of returns to the market shocks.

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The multivariate GARCH models are usually used to analyze the volatilities and co-volatilities across markets (Kearney and Patton, 2000), being designed to quantify the way in which the news is influencing the market volatilities.

Cappiello, Engle and Sheppard (2006) designed an asymmetric version of the DCC model of Engle (2002) in order to examine the degree in which changes in asset correlation show evidence of asymmetric responses to negative returns.

Examining the correlation between the Romanian best known indexes, namely the BET and the BET-FI and the DAX index, could supply a theoretical justification for investors who seek to hedge portfolio exposures. Since correlation is time-varying, we believe that a short-run analysis should point out the degree to which, for instance, the returns volatility increase in a bear market in co-movement with other volatilities. Good estimates of the covariance matrix and correlation structure of the returns are very important for a portfolio manager or for a risk manager. This is the reason why we chose to analyze the short-run returns in 2011, when the European sovereignty debt crisis had spread to all classes of assets.

We are interested to see if the financial uncertainty due to the Greek crisis and low growth environment forecasted for the other European countries in 2011 led to increased correlation among the assets on the Romanian capital market and, if so, whether an asymmetric DCC-GARCH model might supply a better fit, and whether the co-movements in the stock returns were associated with the spread of contagion. The findings will also help us derive conclusions about investors’ keenness to move capital to the Eastern Europe or whether the Romanian capital market is decoupled from the European capital markets.

The paper is structured as follows: section 2 presents a review of the recent literature for multivariate GARCH models and developments related to the DCC models, and in section 3 we present the econometric methodology. In section 4 we describe the data used in the paper, while in section 5 we present the results. Section 6 concludes and discusses areas for further research.

II. Literature Review

Multivariate GARCH models involve the estimation of the covariance matrix which can be made either directly, as in the VEC (Bollerslev et al., 1988), DVEC, and diagonal VEC - a restricted version of VEC, or BEKK (Engle et al., 1995) models or indirectly, using conditional correlations as in CCC, DCC or STCC (Smooth Transition Conditional Correlations) models. The VEC, DVEC and BEKK models face the problem of estimating a very large number of parameters.

The orthogonal GARCH (O-GARCH) proposed by Alexander (2000) represents the errors as linear combinations of uncorrelated factors (similar to principal component analysis), in a smaller number than the error vector in order to reduce the dimension of the covariance matrix. In a highly correlated system, only a few principal components are necessary to describe adequately the returns volatility, and large portfolios may be thus estimated. If the returns are weakly correlated, or the components have similar unconditional variance, problems in the estimation of O-GARCH will occur.
Van der Weide (2002) generalized the O-GARCH model as a GO-GARCH model in order to solve some of the problems. The issue of maximizing the MLE function for larger portfolios led to the development of a three-step procedure (Boswijk and van der Weide, 2006).

A different procedure for the estimation of the GO-GARCH model was developed by Broda and Paolella (2008) by using the independent component analysis (ICA) to reduce the dimension problem to a set of univariate models. The method is called CHICAGO (Conditionally Heteroskedastic Independent Component Analysis of GO-GARCH) and allows for non-normal innovations.

Moreover, in practice it is requested to develop models that take into account multivariate issues such as volatility spillover and contagion effects across markets. The GARCH models are still widely considered models for measuring the financial risk. The interest shown in the class of DCC models is given by the fact that it calculates the correlation between the asset returns as a function of their past volatility and the correlations among them. Coming from the GARCH family, a DCC model uses the recent past information for estimating the present correlation between series. Since correlation may be measured as well as volatility, the estimation of a DCC model is usually achieved in two steps in order to simplify the estimation of the time varying correlation matrix.

The DCC model was introduced by Engle (2002) and its specifications will be discussed in the next section. In comparison with other correlation models, among which scalar BEKK, diagonal BEKK, O-GARCH, the DCC with integrated moving average estimation, the DCC by log likelihood for integrated model and the DCC by log likelihood for mean reverting model, the last one scored better than the others. The selection of the best model was made by using several tests, such as the mean absolute error test, autocorrelation test of the squared standardized residuals and an estimator of the value of risk for two-asset portfolio (Engle, 2002). Similar dynamic correlation models were shortly developed afterwards by Christodoulakis and Satchell (2002) and by Tse and Tsui (2002) – the TVC (Time Varying Correlation) model. A different specification of the Ht is given in the corrected DCC (cDCC) model of Aielli (2009). There is empirical evidence that in practice both models have close results. Anyway, by allowing the standard DCC model to incorporate asymmetries (Cappiello et al., 2006) better results were obtained by modeling the conditional correlations.

The time varying dependence across assets is the copula-GARCH or, more recently, copula-vine approach. The joint distribution function may be decomposed into N marginal distributions and a copula function which describes the dependence between the N assets (see Jondeau (2006) for a copula-GARCH model). Several dynamic copula-GARCH models, which assume that the copula parameters evolve according to a time-varying conditional correlation matrix, were applied to the Romanian, Bulgarian, Polish and Czech stock index returns and the models fitted with skew-t residuals showed better results than the Gaussian or t-residuals models (Acartirnei, 2011). If the margins are normal and the copula is multivariate normal, then the dependence is described by the correlation matrix. If not, then the assumption of multivariate normal distribution is not realistic for modeling asset returns and a copula approach should be used instead.
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Since in practice there are many estimation issues, such as the estimation of randomly chosen subsample, may produce different correlations, or the bivariate estimation, as recommended by many researchers, may give different parameters for correlation, or the dynamics of the returns may have different regimes, the latest models developed for conditional correlations include: a quadratic flexible DCC model (Billio and Caporin, 2009), a generalized DCC model (Hafner and Franses, 2009), a regime switching DCC (Pelletier, 2006), a component DCC (Colacito et al., 2009), STCC model (Silvennoinen and Terasvirta, 2005), a factor-spline-GARCH (Rangel and Engle, 2009), dynamic copula-GARCH and copula-vine models (Aas et al., 2009). Detailed surveys of Multivariate GARCH models are given in Bauwens et al. (2006) and Silvennoinen and Terasvirta (2009).

III. Dynamic Conditional Correlation Models

The study of Cappiello et al. (2006) uses a generalization of the standard DCC model introduced by Engle (2002) and includes the asset-specific correlation of news impact curves and the asymmetric dynamics in correlation.

The asset returns, \( r_t \), are conditionally normal with mean zero, which is a stylized fact, and the conditional covariance matrix, \( H_t \), Following Engle and Sheppard (2001), “the conditional covariance matrix can be decomposed as:

\[
H_t = D_t R_t D_t
\]

where: \( D_t \) is a \( k \times k \) diagonal matrix of time-varying standard deviations from univariate GARCH models and \( R_t \) is the time-varying correlation matrix”.

The multivariate normal distribution was initially assumed in the standard DCC model, but we may model the returns with other distributions. The key element is the correlation matrix, \( R_t \) which is time varying in comparison with the Constant Conditional Correlation model (CCC) in which the correlation is assumed constant, namely \( H_t = D_t R D_t \).

The likelihood of the DCC estimator is:

\[
L = -0.5 \sum_{t=1}^{T} \left( k \log \left( 2\pi \right) + 2 \log \left( |D_t| \right) + \log \left( |R_t| \right) + \varepsilon_t^\prime R_t^{-1} \varepsilon_t \right)
\]

The volatility (\( D_t \)) and the correlation (\( R_t \)) components may vary, thus the estimation process is achieved in two steps. Firstly the volatility (\( L_v \)) is maximized:

\[
L_v = -0.5 \sum_{t=1}^{T} \left( k \log \left( 2\pi \right) + \log \left( |D_t|^2 \right) + \varepsilon_t^\prime D_t^{-2} \varepsilon_t \right)
\]

then the correlation (\( L_c \)) is maximized:

\[
L_c = -0.5 \sum_{t=1}^{T} \left( \log \left( |R_t| \right) + \varepsilon_t^\prime R_t^{-1} \varepsilon_t - \varepsilon_t^\prime \varepsilon_t \right)
\]

(See Engle and Sheppard (2001) for the estimation of the log-likelihood function).
Since the number of the parameters to be estimated is \((n+1)(n+4)/2\) large, Engle proposed a two-step estimation. The DCC model is estimated by a two-step procedure: a) in the first step univariate GARCH models are fitted for each assets' returns and estimates of their variances are thus obtained; b) the returns are filtered out of the GARCH effect (degarched returns) by dividing by their estimated standard deviations and then are used to estimate the dynamics of correlation, 

\[
\varepsilon_{it} = \frac{r_t}{\sqrt{h_t}}.
\]

In the second step, the standardized residuals are used to estimate the time-varying correlation matrix.

The model developed by Engle (2002) has the following non-linear GARCH specification for the conditional correlation:

\[
Q_t = (1-a-b)\bar{Q} + a\varepsilon_{t-1}\varepsilon_{t-1}' + bQ_{t-1}
\]

where \(Q_t = \begin{pmatrix} q_{ij} \end{pmatrix}\) is a \(nxn\) symmetric positive definite matrix, \(a\) and \(b\) are non-negative scalars such as \(a+b < 1\), \(a\) is the news coefficient and \(b\) is the decay coefficient. \(\bar{Q} = \mathbb{E} \begin{pmatrix} \varepsilon_i \varepsilon_j' \end{pmatrix}\) is the unconditional variance matrix of the standardized residuals (the unconditional correlation). The conditional correlations \(q_{ijt}\) are time-varying and follow a structure similar to a GARCH (1, 1) model.

Engle showed that modeling the conditional covariance of the standardized returns is equivalent to modeling the conditional correlation of the returns. For ensuring a conditional correlation between -1 and +1, by normalization the correlation can be expressed as

\[
\rho_{ijt} = q_{ijt} / \sqrt{q_{ii}q_{jj}}.
\]

As in the case of GARCH models, if \(a+b < 1\), the model is mean-reverting, and if the sum of the parameters is equal to 1, then the model is integrated. The correlations are obtained by transforming this to:

\[
R_t = \left( \text{diag}(Q_t) \right)^{-0.5} Q_t \left( \text{diag}(Q_t) \right)^{-0.5}
\]

where \(\left( \text{diag}(Q_t) \right)^{0.5}\) is a diagonal matrix of the square root of the diagonal elements of \(Q_t\).

The limitation of the standard DCC model is the assumption that the conditional correlations follow the same dynamic structure, in contrast to the Markov Switching Model or a Threshold Autoregressive Model where different dynamics may be assumed. If the data have structural breaks, the conditional correlation models may lead to incorrect estimation of the risk. Also, the DCC model is limited to a small number of assets. A GO-GARCH model could simplify computational requirements for large portfolios.

In order to capture the asymmetries in the data, different asymmetric multivariate GARCH models were developed (Cappiello et al., 2006).

The univariate volatility models were selected by the Schwartz Information Criterion (BIC) from the GARCH family capable of capturing the stylized facts of asset returns.
In this respect, we used the following asymmetric models that capture the leverage effect in a different way: the EGARCH model (Nelson, 1991) and the GJR-GARCH model (Glosten, Jagannathan and Runkle, 1993), since GJR and EGARCH allow for threshold effects but use different powers of variance in the variance equation. For each GJR and EGARCH model we modeled the mean equation with an AR(1) and ARMA(1,1) specification.

Because the standard DCC developed by Engle does not include asymmetries, the equation was modified in order to incorporate the asymmetries and asset-specific impact parameters of news.

\[
Q_t = (\tilde{Q} - A\tilde{Q}A - B\tilde{Q}B - G\tilde{N}G) + A'\varepsilon_{t-1}^'\varepsilon_{t-1}^'A + B'Q_{t-1}B + G'\eta_{t-1}\eta_{t-1}'G
\]

where \(A, B, G\) are diagonal parameter matrices, \(n_t = I[\varepsilon_t < 0] \otimes \varepsilon_t\) and \(\tilde{N} = E[\eta_t \eta_t']\). Thus it is computed the Q matrix at time \(t\), given the first lag of Q and the standardized residuals.

We may see that there are four versions of the model:

1) the DCC Garch model if \(G = 0\), \(A = [a_y] = [\sqrt{a}]\), \(B = [b_y] = [\sqrt{b}]\);

2) the Asymmetric DCC Garch model (ADCC) if \(A = [a_y] = [\sqrt{a}], B = [b_y] = [\sqrt{b}], G = [g_y] = [\sqrt{g}]\);

3) the Generalized Diagonal DCC Garch model (GDDCC) if \(G = 0\);

4) the Asymmetric Generalized Diagonal DCC Garch (AGDDCC) model was developed to capture the heterogeneity in returns, so that it allows for different news impact and smoothing parameters across the assets (for more information see Cappiello, Engle and Sheppard, 2006).

IV. Data

The data used in this paper are the daily closing prices of two Romanian stock exchange indexes, namely BET and BET-FI, and the DAX index in 2011. The data concerning the Romanian indexes are available from the Bucharest Stock Exchange website and the data concerning DAX are available at the yahoo website. All data are denominated in euro. We did not use pseudo-closes, namely sampling the prices at the same GMT, but the closing prices. There are 250 observations from January 4, 2011 to December 23, 2011.

Considering their properties, we may see that the data have the properties usually noticed in the case of financial returns: the returns are leptokurtic, have negative
skew, and extreme excess kurtosis. Generally, by standardizing the returns, they can be normal or close to normal. To investigate the properties of innovations, we standardized the residuals in every GARCH model. The residuals obtained were less skewed and less fat-tailed, but they were still non-normal, rejecting the Jarque-Bera test at 1% level. Therefore, the univariate GARCH models applied to the stock index returns were modeled with a Student-t distribution.

V. Empirical Results

We modeled each time series with an EGARCH and GJR model, using a Student-t distribution, while for the mean equation we used an AR(1) and ARMA(1,1) specification.

<table>
<thead>
<tr>
<th></th>
<th>AR(1)-EGARCH(1,1)-t</th>
<th>ARMA (1,1)-EGARCH(1,1)-t</th>
<th>AR(1)-GJR(1,1)-t</th>
<th>ARMA(1,1)-GJR(1,1)-t</th>
</tr>
</thead>
<tbody>
<tr>
<td>BETFI</td>
<td>617.4954</td>
<td>617.5381</td>
<td>623.1569</td>
<td>622.9330</td>
</tr>
<tr>
<td>BET</td>
<td>726.2707</td>
<td>727.1098</td>
<td>729.4941</td>
<td>733.2530</td>
</tr>
<tr>
<td>DAX</td>
<td>678.8595</td>
<td>678.8878</td>
<td>686.3899</td>
<td>686.6053</td>
</tr>
</tbody>
</table>

Table 1 presents the log likelihood of the univariate GARCH models estimated for BETFI, BET and DAX stock index returns. We see that, although the models are different in specifications, they all come very close, with a little improvement for the AR(1)-GJR(1,1) with Student-t distribution. We estimated the same models with the normal distribution, but the models estimated with the Student-t distribution showed significantly better results.

We have an expectation that the returns should show some significant correlation in August and November, 2011 since a lot of events happened at that time. We mention only some which occurred in August 2011: some of the biggest drops in stock prices in the USA, Europe and Asia due to fear of contagion of the European debt crisis towards Spain and Italy in the first place. A reform of the Spanish constitution was necessary in August for winning back market confidence. S&P downgraded America' credit rating from triple A to AA+. Many important stock exchanges faced severe declines: CAC40 fell by 20% in two weeks, the DAX fell by 5.8% in one day on 18 August and FTSE100 fell by 4.5% also on 18 August. Also, widespread fears about the reliability of the Greeks banks were ever-present and talks about the stability of the Euro Zone led to a higher volatility across the stock markets.

Figures 1-3 depict four models for dynamic correlations between BETFI and BET, BET and DAX, BETFI and DAX. The stock index returns were degarched using only the AR(1)-EGARCH(1,1)-t and AR(1)-GJR(1,1)-t models, since some of the ARMA coefficients were not statistically significant.
The dynamic correlations of DCC and A-DCC models are almost the same for the EGARCH model, while the AGD-DCC (GJR) model shows a different dynamics, with greater amplitude in comparison with DCC, A-DCC and GD-DCC, which follow a
similar pattern. The GD-DCC and AGD-DCC show a similar pattern when using the EGARCH specification. All models show a significant correlation between BETFI and BET, within the [0.5, 0.9] interval. The highest correlation was reached in August, thus showing that the AGD-DCC model was more sensitive to incorporate negative news than the other models.

The correlation between BET and DAX becomes significant in the second semester, with the momentum of the European crisis for both specifications. We may notice a clear spike in the GD-DCC model with EGARCH specification in August. The AGD-DCC and GD-DCC are more volatile than the others. The GD-DCC and A-DCC models, with GJR specification, capture the spillover from Greek events in November.

**Figure 3**

Dynamic correlations between BETFI and DAX

We see the same seesaw pattern as before for the GD-DCC and AGD-DCC models. There are spikes of correlation that are significant, over 0.5, corresponding mainly to the influence of external factors such as the escalation of the Greek crisis.

We decided to answer the question which the best DCC-GARCH model is to estimate the Romanian capital market risk seen in interdependence with other capital markets.

According to the BIC information criteria, we selected individual GARCH models which capture leverage in the variance equation and with autoregressive and moving average terms for the mean equation.

The best model seems to be the AGD-DCC, an asymmetric generalized diagonal DCC model that does include asymmetries. Very close to it is the GD-DCC model. We see in the above figures that both models are capable of capturing spillovers from other capital markets, while the other models, although close to them, are not so responsive.
Our intuition was that the conditional equity correlation is bound to increase when bad news affect the financial markets. For this reason, the class of asymmetric models should provide a better model for the conditional correlation. It remains to test in other paper how they respond to the positive news and whether they produce better forecasts than the non-asymmetric models. Therefore, we should see if the asymmetric models suffer from a lack of additional effectiveness because of potential misspecification of the univariate GARCH models employed or to accumulation of estimation errors because of a larger number of the model parameters.

Table 2

<table>
<thead>
<tr>
<th>Log-likelihood</th>
<th>AR(1)-EGARCH(1,1)-t</th>
<th>AR(1)-GJR(1,1)-t</th>
</tr>
</thead>
<tbody>
<tr>
<td>DCC</td>
<td>2134.9823</td>
<td>2149.6994</td>
</tr>
<tr>
<td>A-DCC</td>
<td>2134.9823</td>
<td>2150.1284</td>
</tr>
<tr>
<td>GD-DCC</td>
<td>2144.5551</td>
<td>2155.2640</td>
</tr>
<tr>
<td>AGD-DCC</td>
<td>2146.4343</td>
<td>2160.2817</td>
</tr>
</tbody>
</table>

The peak of the European sovereign debt crisis and the biggest drops in equity across European stock markets can be located in August, while the other spell of uncertainty was in November 2011. All dynamic correlations for BETFI and BET show that their highest correlation occurred in August, indicating that the main indexes of the Romanian capital market responded to external events and that the AGD-DCC models can incorporate volatility spillovers. The results indicate that there is a significant increase in the co-movements of stock index returns, thus indicating the spread of contagion on the Romanian capital market.

Figure 4

An AGD-DCC model with EGARCH/GJR specifications
Other tests may prove useful in order to determine false periods of significant correlation. Figure 4 shows how different the results may be, even if we use close specification for GARCH univariate processes. The AGD-DCC model with GJR specification has a seesaw pattern, with correlations becoming significant only in the second half of 2011. The other model has a more steady dynamics, implying a more stable dynamics between BET and DAX indexes, although insignificant in the first semester, but converging towards the dynamics of the AGD-DCC (GJR) model.

VI. Conclusions

We intended to study asymmetric DCC-GARCH models capable of identifying volatility spillover and contagion effects across capital equity markets. The current international financial turmoil revealed a high interdependence between the capital equity markets, as high volatilities were recorded simultaneously on the international stock markets.

In this paper, we investigated if there are any volatility spillovers from developed capital markets and contagion effects, namely between the Romanian capital market and the European capital market, taking the German index (DAX) as a proxy, given its importance for other financial markets.

In this respect, we used the daily returns of the main stock indexes of these markets, BET and BETFI for the Romanian capital market and the DAX index, observed in 2011, in order to investigate the short-run dynamics correlation between them.

Following Cappiello et al. (2006) we employed four DCC models, namely the DCC Garch model (DCC), the Asymmetric DCC Garch model (A-DCC), the Generalized Diagonal DCC Garch model(GD-DCC), and the Asymmetric Generalized Diagonal DCC Garch (AGD-DCC) model. Out of the four models, two were asymmetric.

There is evidence that the AGD-DCC model is more sensitive to negative news than the other DCC models, while having the best fit irrespective of the GARCH specification used, that is AR(1)-EGARCH(1,1) and AR(1)-GJR(1,1) with Student-t distribution. Other GARCH specifications should also be tested.

We noted that the conditional correlations of the BET index and the DAX index considerably increased during the crisis period, namely in the second semester of 2011, when the European debt sovereign crisis reached its peak.

There is evidence to conclude that during the crisis the volatility spillovers and contagion effects were significant, but not very strong between the Romanian and the German capital markets. Therefore, we may say that the Romanian capital market responds to some extent to external influences. It remains to be tested in other paper how indices respond to positive news and whether the asymmetric DCC models produce better forecasts than the non-asymmetric models.

The model results agree with the conclusions of Cappiello et al. (2006), namely that the log-likelihood of the models increases when we include asymmetric effects.

At last, we would like to suggest that studying dynamic conditional correlations between markets is a practical endeavor, at many different levels, from developing Value at Risk estimation for portfolio managers who need to hedge their portfolio exposure to financial risk, to designing better risk indicator tools for risk manager
officers and also for capital market authorities, for assessing the impact of volatility spillovers from other markets.

References


