4. HAS NONLINEARITY RESOLVED THE ANOMALY OF UNIT ROOT BEHAVIOUR IN FORWARD DISCOUNT? NEW EMPIRICAL EVIDENCE

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Abstract

In this study we analyse the issue of mean reversion in forward discount based on nonlinear framework for seven currencies. Compared to previous study, we apply a novel approach of a threshold regression (TAR) and followed by nonlinear unit root tests. This approach disentangles tbodhe issue of nonlinearity and nonstationarity in forward discount. After applying nonlinearity test, we found evidence of nonlinearities in five out of seven currencies for the forward discounts. Notably, it is found that forward discount behaves as unit root in a band and becomes mean reverting outside the band. This explains the mixed findings in earlier studies due to general assumption of linearity in forward discount.

Keywords: forward discount, threshold autoregression (TAR), forward bias puzzle, efficiency

JEL Classification: F30, F31, C22, C53

1. Introduction

One of the challenging aspects in understanding forward rate unbiasedness hypothesis (FRUH) is unravelling the time series property of forward discount. Researchers in this field have yet to come up with a consensus on how to model this variable. The difficulties involved in modeling forward discount can be seen from it's diverge properties that exist in the variable. Previous findings have found that forward discount is very persistent, and there are mixed evidences of mean reversion, long memory[2] and spurious long memory due to structural breaks.[3]

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[2] *This has led to research that argues forward discount is fractionally integrated (e.g., Baillie and Bollerslev , 1994).*

There are three reasons why determining whether forward discount is mean reverting or not are important. Firstly, the time series properties of forward discount are directly related to risk premium. If one founds that forward discount is nonstationary, similar property will also exist in risk premium. The main contention is that to apply theoretically a nonstationary risk premium is really difficult (Crowder (1994)). Furthermore, it will also invalidate the idea of error correction term in cointegration of nominal spot exchange rate as a proxy of risk premium as suggested by Crowder (1994). Failure to become the proxy of risk premium will lead to rejection of efficient market hypothesis (EMH) where cointegration implies predictability.

Secondly, based on the theory of covered interest parity (CIP), it is stated that when CIP holds, the forward discount is simply the difference between the domestic and foreign interest rate. Since interest rate is bounded below by zero, the linear combination of this rate cannot have unbounded mean, which is a true unit root process. Regardless of whether CIP holds, understanding of forward discount is crucial since CIP states that forward discount and interest differential should share the same order of integration.

Finally, nonstationarity of forward discount will invalidate the standard statistical inference of regression based test which FRUH testing is based on. This will lead the regression based test of FRUH to become unbalanced. Thus, anomalous findings in FRUH may partly be explained due to the finding of unit root in the forward discount. These highlight the importance of understanding the time series properties of forward discount, especially in determining whether it is mean reverting or not.

Various unit root test procedures have been applied in testing forward discount, with different frequencies and time periods, and yet it is still far from conclusive.[4] It is a well-known fact that unit root test is not good at distinguishing a series with characteristic root that is close to unity[5] and structural change. In the case of structural change, the standard Dickey-Fuller test is biased towards the non-rejection of a unit root. However, allowing for multiple breaks blurs the distinction between a unit root process and stationary series with breaks (Hansen, 2001) and the actual test creates difficulties of practical implementation (Perron, 2006).

However, previous empirical research in analysing mean reverting of forward discount relies on the assumption that forward discount is linear. There are several reasons that we might suspect that forward discount is nonlinear. One of the reasons is the existence of transaction cost. This idea is pioneered by Dumas (1992), where he developed a general equilibrium model of exchange rate determination in spatially separated markets with significant cost of international trade. Based on the model, the transaction cost will create a band of inaction, where inside the band; there is no adjustment in deviation from equilibrium that takes place. However, outside the band, the process becomes mean reverting since the benefits of arbitrage exceed the cost.

The idea of limits to speculation hypothesis of Lyons (2001) may also explain nonlinearities in forward discount. The model emphasizes on the importance of

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[3] *For near unit root see Crowder (1994) , Evans and Lewis (1995), long memory see Baillie and Bollerslev (1994), Maynard and Phillips (2001) and for structural break see Sakoulis and Zivot (2001) and Sakoulis et al. (2010).*

[4] *See Engle (1996) for an excellent survey.*

[5] *This is exactly the case of forward discount*

Sharpe Ratio in determining the investment strategies.[6] It states that when the Sharpe Ratio is higher than a threshold level, the deviation from uncovered interest parity (UIP) will be high enough to be viewed as arbitrage opportunity. The implication of the model is that it creates a band of Sharpe Ratio which results in a band of forward discount where UIP does not hold.[7] Other possible explanations of nonlinearities of forward discount are brought forward by Sarno *et al.* (2004), which are automatic trading rules, heterogeneous belief and tendency of traders to wait for large arbitrage opportunities before entering the market.

Although evidence of nonlinearities in foreign exchange rate has been found by previous studies,[8] it does not distinguish between nonlinearity from unit root behaviour in the exchange rate. One important aspect is that if prior assumptions of stationarity are not valid and the variables have unit root, it will lead to incorrect inferences of the test of linearity versus threshold alternatives. This is due to the nonstandard asymptotic distribution of the test.[9] A decade ago, Caner and Hansen (2001), afterwards CH, has developed a symmetric threshold autoregressive (TAR) model with an autoregressive unit root. CH is the first to combine the presence of unit root type of nonstationarities and threshold type of nonlinear dynamics. Their major contribution was the development of a new asymptotic theory for detecting the presence of threshold effects in a series which was restricted to be a unit root process under the null of linearity. In this model, we allow for general autoregressive orders and we do not restrict the coefficients across regime.

The remainder of the paper is organized as follows. In section 2, we discuss the idea of FRUH and followed by methodology in section 3. Description of data are reported in section 4 followed by findings in section 5 and 6. The last section concludes the paper.

2. FRUH and Stationarity of Forward Discount

The uncovered interest parity (UIP) can be stated as:

$$i_t = i_t^* + \Delta s_{t+1}^e \qquad (1)$$

where: s_t represents logarithm of spot exchange rate, i_t and i_t^* represents local and foreign nominal interest rate of similar securities respectively, and superscript e denotes market expectation based on information at time t . In testing the efficiency of foreign exchange market, researchers mostly focus on the relationship between spot and forward exchange rate. This approach is possible with the assumption that covered interest parity (CIP) holds. CIP can be stated as:

$$i_t = i_t^* + (f_t - s_t) \qquad (2)$$

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[6] *Sharpe Ratio is defined as $\frac{E[R_s - R_{rf}]}{\sigma_s}$ where $E[R_s]$ is the expected return on the strategy, R_{rf} is the risk-free interest rate and σ_s is the standard deviation of the returns of the strategy.*

[7] *Some researchers used UIP testing instead of FRUH testing. The words are used interchangeably in research.*

[8] *See Baum et al. (2001), Clarida et al. (2003), Kilian and Taylor (2003), Panos et al. (1997), Obstfeld and Taylor (1997), Sarantis (1999) and Taylor et al. (2001).*

[9] *See Bec et al. (2002)*

where: f_t is the logarithm of forward rate and $(f_t - s_t)$ is forward discount(premium). A vast amount of empirical evidences suggest that CIP indeed holds empirically.[10]. By combining equation (2) and (1) above, it can be stated that:

$$i_t = i_t^* + \Delta s_{t+1}^e \tag{3}$$

which leads to a popular approach in testing the forward rate unbiasedness hypothesis in the form of:[11]

$$\Delta s_{t+1} = \beta_1 + \beta_2 (f_t - s_t) + \varepsilon_{t+1} \tag{4}$$

where: ε_{t+1} is a rational expectation forecast error. The hypothesis of unbiased forecast of the forward rate and efficient market hypothesis are based on $\beta_1 = 0$, $\beta_2 = 1$ and $E_t[\varepsilon_{t+1}] = 0$. This shows that the forward discount or $(f_t - s_t)$ is unbiased predictor of the future currency depreciation or Δs_{t+1}. However, as noted by Engel (1996), the finding of $\beta_2 < 0$ is robust. This is however unacceptable, since the more the foreign currency is at discount, the more home currency is to appreciate rather than to depreciate to offset the interest differential. This phenomenon is best known as forward bias puzzle.

An interesting aspect in equation (4) is the behaviour of forward discount. In order for the above regression to be balanced, both of the variables in the above regression must possess the same order of integration. It is a well-established fact that nominal exchange rate behaves like $I(1)$ process, thus Δs_{t+1} behaves like $I(0)$. As noted by Engel (1996), if the order of the integration of $(f_t - s_t)$ is between -0.5 and 0.5, then forward discount is mean reverting and the estimate of β_2 above is consistent. Otherwise, if the order of integration of $(f_t - s_t)$ is > 0.5, forward discount is non-stationary, which results in the estimate of β_2 to be inconsistent.

The existence of risk premium can be implied as:[12]

$$f_t - s_t = \Delta s_{t+1} + \xi_t + \varepsilon_{t+1} \tag{5}$$

with the assumption that the agents are risk averse and rational expectation holds where ξ_t is time-varying risk premium. With the assumption of rational expectation which leads to stationarity of ε_{t+1} and robust finding of stationarity in exchange rate depreciation, Δs_{t+1}, it shows that time series properties of forward discount are directly related to the risk premium.

1. Nonlinear Unit Root Test Model[13]

The model is based on threshold autoregression (TAR) of the following:

$$\Delta fd_t = \theta_1' x_{t-1} 1_{\{z_{t-1} < \lambda\}} + \theta_2' x_{t-1} 1_{\{z_{t-1} \geq \lambda\}} + e_t \quad t = 1, \dots T \tag{6}$$

where: fd_t represents forward discount, $x_{t-1} = (fd_{t-1}, 1, \Delta fd_{t-2}, \dots, \Delta fd_{t-k})$, $1_{\{\cdot\}}$ is an indicator function and e_t is i.i.d. error term. The threshold variable,

[10] *See Sarno et al. (2003).*

[11] *Also popularly known as Fama regression based on his paper of Fama (1984).*

[12] *See Fama (1984) for detail discussion.*

[13] *This section draws heavily on Caner and Hansen (2001). Interested reader can refer to the paper for further details.*

$Z_{t-1} = fd_{t-1} - fd_{t-m-1}$, with m as the delay parameter of $1 \le m \le k$ The λ represents the threshold value and is unknown, and takes on value in the compact interval $\Lambda = [\lambda_1, \lambda_2]$ where λ_1 and λ_2 are picked so that $P(Z_{t-1} \le \lambda_1) = .15$ and $P(Z_{t-1} < \lambda_2) = .85$. The threshold variable $Z_{t-1} = fd_{t-1} - fd_{t-m-1}$ specification is necessary due to the econometric theory developed does not allow levels form. The components of θ_1 and θ_2 can be separated into:

$$\theta_1 = \begin{pmatrix} \rho_1 \\ \beta_1 \\ \alpha_1 \end{pmatrix}, \theta_2 = \begin{pmatrix} \rho_2 \\ \beta_2 \\ \alpha_2 \end{pmatrix} \tag{7}$$

where: ρ_1, ρ_2 are the slope on fd_{t-1}, (β_1, β_2) represents the intercept coefficients and $(\alpha_1 1, \alpha_1 2)$ are the slope on dynamic regressors of $(\Delta fd_{t-1}, \dots, \Delta fd_{t-k})$.

The model (6) is estimated by least square (LS) of;

$$\Delta fd_t = \hat{\theta}_1'(\lambda) x_{t-1} 1_{\{Z_{t-1} < \lambda\}} + \hat{\theta}_2'(\lambda) x_{t-1} 1_{\{Z_{t-1} \ge \lambda\}} + \hat{e}_t(\lambda) \tag{8}$$

where: $\hat{e}_t(\lambda)$ is the LS residuals an $\hat{\sigma}^2(\lambda) = T^{-1} \sum_{t=1}^{T} \hat{e}_t(\lambda)^2$ denotes the residuals variance. The threshold parameter is estimated by minimizing $\sigma^2(\lambda)$:

$$\hat{\lambda} = \arg\min_{\lambda \in \Lambda} \hat{\sigma}^2(\lambda) \tag{9}$$

The above optimal threshold $\hat{\lambda}$ is then plugged into equation (8) to determine the other parameters of interest. The first test statistic is to test the existence of nonlinearity in the series due to threshold, where the threshold effects disappear under the joint hypothesis

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The above optimal threshold $\hat{\lambda}$ is then plugged into equation (8) to determine the other parameters of interest. The first test statistic is to test the existence of nonlinearity in the series due to threshold, where the threshold effects disappear under the joint hypothesis of:

$$H_0: \theta_1 = \theta_2 \quad (10)$$

The test statistics is:

$$\underset{\lambda \in \Lambda}{\sup} W_t(\lambda) = \underset{\lambda \in \Lambda}{\sup} T\left(\frac{\hat{\sigma}_0^2}{\hat{\sigma}^2(\hat{\lambda})\lambda} - 1\right) \quad (11)$$

where: $\hat{\sigma}^2$ and $\hat{\sigma}_0^2$ are the residuals variances from the TAR and linear AR models respectively.

Based on model (6), parameter (ρ_1, ρ_2) determines the stationarity of the process, which result in three different hypothesis of:

$$H_0: \rho_1 = \rho_2 = 0 \quad (12)$$

a unit root process of forward discount,

$$H_0: \rho_1 < 0 \text{ and } \rho_2 < 0 \quad (13)$$

two-regimes stationary threshold autoregressive AR and

$$H_2: \begin{cases} \rho_1 < 0 \text{ and } \rho_2 = 0 \\ \quad or \\ \rho_1 = 0 \text{ and } \rho_2 < 0 \end{cases} \quad (14)$$

a partial unit root case. If the H_2 hypothesis holds, the process of fd_t behaves like a unit root process in one regime, yet behaves as mean reverting process in the other regime. Thus, the process under H_2 process is nonstationary, but it is not the classical unit root process (Caner and Hansen, 2001).

Since H_1 hypothesis is one-sided, the Wald's one-sided test statistic of $H_0: \rho_1 = \rho_2 = 0$ vs H_1 is:

$$R_{1T} = t_1^2 1_{\{\hat{\rho}_1<0\}} + t_2^2 1_{\{\hat{\rho}_2<0\}} \quad (15)$$

where: t_1 and t_2 are the t-ratios of the $\hat{\rho}_1$ and $\hat{\rho}_2$ respectively of equation (8). However, in order to discriminate between H_1 and H_2, we cannot rely on R_{1T} statistics above (Caner and Hansen (2001)). As suggested by Caner and Hansen (2001), in order to test H_0 vs H_2, we rely on the negative of the $-t_1$ and $-t_2$ statistics.

4. Data And Unit Root Test

In this study, seven currencies, i.e. the Australian dollar, the Canadian dollar, the Danish krone, the Japanese yen, the Norwegian krone, the Swedish krona and the United Kingdom pound sterling for the period from January 1997 to January 2011 are used. Monthly data is obtained from Datastream and quoted in USD terms. The construction of forward discount is through subtracting 1-month log of forward rate with log spot rate and multiplying it with 100 resulting in series that is quoted in percentage per month basis[15].

In Table 1, we report the results of the Augmented Dickey-Fuller (ADF) test, where the number of lag is determined by the sequential t -test.[16] All of the tests are conducted with intercepts. The nullity of unit root is not rejected for most of the currencies forward discount, except for Canada and Norway. Based on the ADF test, mean reverting process of forward discount is uncertain. This finding is in line with previous research, where mixed results are reported. It is a well-known fact that the ADF test is sensitive to misspecification and has low power against a stationary alternative with high level of persistency. Furthermore, as noted by Taylor (2001), Taylor and Peel (2000) and Taylor *et al.* (2001), if the data-generating process (DGP) is indeed nonlinear, with linear test specification, the ADF will be biased towards failing to reject the null hypothesis.

5. The Linearity Test

In Table 2, we report the result of the Wald's test of equation (11), testing for nonlinearity in forward discount due to threshold. The critical values of the test are based on 10,000 bootstrap simulations as suggested by CH. It also has excellent size and good power in small sample (Caner and Hansen, 2001). The optimal delay parameters m are determined endogenously, where it minimizes the error sums of squares of the TAR model. The larger the value of m, the longer it takes for agents to react towards deviation from the relationship that link spot and forward rates. For example, the arbitrage opportunity for Australia, Canada, Sweden and UK are exploited rapidly where the value of m is one month.

As reported in Table 2, the results show that most of the forward discount of currencies supports nonlinearity. The existences of threshold at 5% significant level are found in five currencies, with exception of Sweden and United Kingdom.[17] Overall, forward discount is inherently nonlinear which suggest misspecification in the functional form of ADF regression test earlier. This might explain our finding of unit root process in most of the currencies earlier where nonlinearities will cause ADF test to be biased towards failing to reject the null hypothesis.

6. The TAR Unit Root Test

Based on the delay parameter selected by the Wald test in Table 2, we report the one-sided Wald statistics of R_{1T} along with t_1 and t_2 to determine the stationary property of

[15] See equation (2).

[16] This technique is suggested by Basci and Caner (2005) due to non-changes of lag order under the null and various alternatives. With maximum of 12 lags, lags that are insignificant are dropped until we reach a t -stats around $|2|$.

[17] For Sweden and United Kingdom, even though the test failed to reject the null hypothesis, the p-value suggest borderline non-rejection of the null at 10% significant level.

forward discount. Specifically, the test statistic of t_1 and t_2 are to test H_0vs H_2, while R_{1T} is to discriminate between H_0 vs H_1. The bootstrap p-value for each statistics in Table 3 are obtained using 10,000 bootstrap simulations corresponding to m delay parameters reported in Table 2.

As Table 3 shows, the R_{1T} statistics shows four currencies are two-regime stationary nonlinear model except for Canada. However, further investigation based on individual t-ratios for Canada, the inside band has unit root whereas the outside band has mean reverting behaviour. This may suggest that the first regime (inside band) is dominant for Canada. Overall, based on R_{1T}, t_1 and t_2 statistics, all the five currencies shows strong evidence of partial unit root process where forward discount has unit root inside the band and mean reversion outside the band. Thus, our findings of unit root process for forward discount in Table 1 have to be viewed with caution. This also implied to the previous findings of nonstationarity in forward discount where it might be a partial unit root process.

The size of the roots ρ_1 and ρ_2 reported in Table 3 suggest that forward discount is more persistent process for inside band as compare to outside band. The TAR regression function is splits depending whether the change in forward discount lie above or below the threshold value estimated in Table 3. Focusing on Australia, the TAR regression function is splits if the 1 month change of forward discount is above 0.0378 or otherwise. The inside band of Australia behave as random walk with drift with 84.3% of observations, while the outside band is a stationary process with 15.7% of observations. Generally, forward discount is shown to be globally stationary although it is very persistent since most observations lie inside the dominant unit root regime of inside band.

7. Conclusion

Our study has provided a new approach in understanding the issue of mixed finding of mean reverting in forward discount as reported in previous studies. The present study is designed to determine the stationarity of forward discount if we allow the series to be nonlinear. Since nonstationary forward discount is hardly acceptable theoretically, few arguments have allowed for nonlinearity in forward discount, which might explain the mixed finding in previous studies as standard unit root test did not take into account the issue of nonlinearity.

Our findings conclude that in general, forward discount is shown to be globally mean reverting process although it is very persistent. The forward discount is very persistent since most observations lie inside the dominant unit root regime of inside band. The findings of threshold nonlinearity and high persistency might explain why the standard ADF tests are biased towards the unit root null hypothesis.

The evidence from this study suggests that nominal interest rate differential between domestic and foreign country will be a partial unit root process. Furthermore, this finding has major implication towards the distribution of the standard foreign unbiasedness hypothesis that previous studies have relied on. Overall, these findings enhance our understanding of the forward bias puzzle that has dominated the field of international finance.

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Appendix

Table 1

Unit Root Test							
	Australia	Canada	Denmark	Japan	Norway	Sweden	United Kingdom
Lag	2	5	10	2	4	5	1
ADF	-1.1793	-2.7380	-2.4034	-0.8324	-2.7859	-2.3731	-1.9441
	[0.6829]	[0.0699]	[0.1424]	[0.8069]	[0.0625]	[0.1510]	[0.3115]
ρ_1	-0.0237	-0.0818	-0.0984	-0.0193	-0.0421	-0.0457	-0.0730
	(0.0201)	(0.0299)	(0.0409)	(0.0232)	(0.0151)	(0.0193)	(0.0375)
1) 10%, 5% and 1% critical value are -2.5764, -2.8795 and -3.4715, respectively,							
2) Lag length is based on sequential t-test.							
3) Figure in parentheses and brackets are standard errors and p-values, respectively,							

Table 2

Threshold Test						
	m	Wald	p-value	10% BCV	5%BCV	1%BCV
Australia	1	21.1	[0.0389]	16.3	19.8	28.9
Canada	1	38.3	[0.0164]	24.3	29.3	41.9
Denmark	10	136.0	[0.0052]	53.8	71.0	122.0
Japan	9	127.0	[0.0027]	53.1	67.2	99.9
Norway	4	41.3	[0.0025]	19.6	22.9	32.0
Sweden	1	24.0	[0.1240]	25.9	32.8	49.7
United Kingdom	1	15.1	[0.1050]	15.3	19.4	31.8
1) m is the optimal delay parameter.						
2) Bootstrap critical value (BCV) is based on 10,000 replications.						

Table 3

TAR Unit Root Test									
	m	t_1	t_2	R_{1T}	λ	%		ρ_1	ρ_2
		Inside band	Outside band	Unit root test	Threshold/ band	Inside band	Outside band	Inside band	Outside band
Australia	1	[0.8010]	[0.0249]	[0.0667]	0.0378	84.3	15.7	0.0014 (0.0208)	-0.1980 (0.0530)
Canada	1	[0.4020]	[0.0746]	[0.1280]	0.0252	83.4	16.6	-0.0436 (0.0313)	-0.1650 (0.0562)
Denmark	10	[0.3970]	[0.0171]	[0.0355]	0.1600	84.2	15.8	-0.0476 (0.0320)	-0.7050 (0.1410)
Japan	9	[0.3280]	[0.0333]	[0.0605]	0.1710	84.5	15.5	-0.0348 (0.0216)	-0.4710 (0.1230)
Norway	4	[0.1570]	[0.0002]	[0.0012]	0.1220	84.1	15.9	-0.0351 (0.0145)	-0.3310 (0.0583)
1) m is delay parameter.									
2) Figure in parentheses and brackets are standard errors and p-values, respectively.									