3. Connectivity-based Clustering of GDP Time Series[1]

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Abstract

The paper presents an alternative connectivity relationship in the form of band limited, sub-quarterly time shifts between quarterly sampled series of the variations of the gross domestic product (GDP). Eight groups around the GDP were identified, expressed in the most powerful European currencies, the Euro (EUR), the Swiss Franc (CHF), and the Great Britain Pound (GBP). The findings allow for building time maps with daily resolutions that endorse the deterministic perspective of the chronological spread of financial fluctuations generated by bilateral trades. The German GDP seems to impose the timing of the aggregated GDP of the Eurozone-12. The method is helpful to detect time delays between processes in the range of 2-4Q, so that knowing the sense and the magnitude of the movements in the advanced series might indicate the trends in the delayed one, or to lay the input variables in correct time positions in multivariate autoregressive lag models or in mixed frequency models.

Keywords: time shift, cluster, time map, bilateral trade, financial fluctuations
JEL Classification: C33; C53; E01; F16

I. Introduction

Detecting connections among the processes embedded in synchronously sampled quantities in the form of time series may help the development of techniques for economic and financial forecasting. Forecasting models such as the Dobrescu model (Dobrescu, 2014) are generally based on information acquired from macro-economic indicators that reveal the historical evolution of various economic processes available in the form of time series. Since the economies are interconnected via trading, financial, and information flows, it is natural to look for connections among indicators and investigate possible regressive multivariate relationships. In this respect, the forecasting techniques could be significantly improved by grouping the series with similar features in the time domain (shape variation), in the frequency domain (coherence), or combined. Such similar groups, or clusters, help to reveal synchronization among trading processes (Păuna *et al*., 2014), comovements of business cycles (Mumatz *et al*., 2011),

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to explain the sensitivity to regional and global crises (Dedu *et al*., 2010; Preda and Mihăilescu, 2009), to test the economic globalization (Miśkiewicz and Ausloos, 2010; Gligor and Ausloos, 2008; Ausloos and Lambiotte, 2007), or to examine the dynamics of stock markets (Aste *et al*., 2010).

Generally, two independent processes are uncorrelated and randomly offset one another. The clustering techniques based on connectivity relationships help to acquire information about the future evolution of a particular process based on similar features it shares with the neighbors in the group. According to Pascual-Marqui (2007), a connectivity relationship could be found by investigating the phase synchronization involved in the complex cross coherence function (CCCF). Phase synchronization measures in the frequency domain were used in pattern recognition of yearly GDP time series (Maharaj *et al*., 2010) or to separate the long-run correlations from the short-run ones among distinct processes (Zhang *et al*., 2011).

The present paper proposes several new approaches. Firstly, a more restrictive and, consequently, a stronger connectivity relationship in the form of constant temporal shifts embedded in coherent series across the 2-4Q band. Here, the phase synchronization is not enough for connection, but only a prerequisite to further investigate the existence or the lack of significant temporal shifts. The theoretical undertaking is supported by the following reasons: i) any effect is more or less a replica of its source; therefore, the cause and the effect should be coherent or correlated; ii) influences need time to show their effects by propagation or diffusion; therefore, the effect should follow the cause; iii) economic processes are quasi-periodic (seasonal variations, business cycles, daily trades); therefore, the influence should be revealed not at a unique frequency but in a band, particularly in the upper 2-4Q band when focusing on the point-to-point variation of GDP; iv) economic processes are rationally planned; therefore, the influence should be stationary during a characteristic time. If time shifts between the GDP series were identified, then their underlying short-run processes are connected and they can be grouped together. Such results could complement the Granger causality tests or clarify the reverse causation issues. Secondly, phase synchronization is evaluated here using the most sensitive part of the CCCF to bilateral variations instead of the squared coherency or the dynamic coherence widely proposed in literature (Croux *et al*., 2001). Thirdly, different from the usual case of detecting time shifts that are multiples of one sampling step, the method proposed here is effective to estimating time gaps smaller than the sampling interval. This is useful when forecasting the values of macro-economic indicators before they become publicly available in the early GDP forecasting models (Sinclair and Stekler, 2013), or when working with heterogeneous data sets from the perspectives of the sampling rate in the bridge and factor models (Foroni and Marcellino, 2013), where it is mandatory to lay the input variables in correct time positions (Zheng and Rossiter, 2006). The method can also be useful to measure propagation delays in diffusion or contagious processes, or in the case of speculative trades on the stock exchange market, where any trader tries to predict the competitors' sell/buy actions by looking for the fingerprint of their decision taking. Fourthly, we preferred here a more straightforward approach to establishing overlapping clusters characterized by the local clustering coefficients (Watts and Strogatz, 1998) instead of the common representation based on c-means algorithms and the associate dendrograms (Bezdek, 1981).

The organization of the paper is as follows: Section 2 explains in brief the principles of the method, Section 3 provides information about data, Section 4 presents the clusters and the time mapping with daily resolution. Finally, Section 5 summarizes the conclusions.

II. Theory and Method

II.1. Phase Synchronization

As it often happens in econophysics, the method is using the analogy between waves and economic processes revealed in the form of time series (Granger and Newbold, 1986). Denoting by $x(t)$ and $y(t)$ two time series, and by $X(f)=|X(f)|\cdot e^{\mathbf{i}\varphi_x(f)}$ and $Y(f)=|Y(f)|\cdot e^{\mathbf{i}\varphi_y(f)}$ their Fourier images, then the CCCF is:

$$\gamma_{xy}(f)=\left|\gamma_{xy}(f)\right|\cdot e^{\mathbf{i}\alpha_{xy}(f)}, \qquad \mathbf{i}=\sqrt{-1}. \tag{1}$$

Next, denoting the phase difference by

$$\Delta\varphi(f)=\varphi_x(f)-\varphi_y(f), \tag{2}$$

then the absolute value of CCCF is the coherency, or the coherence function (CF)

$$\left|\gamma_{xy}(f)\right|=\frac{\sqrt{\left[<|X(f)|\cdot|Y(f)|\cdot\cos(\Delta\varphi)>_\tau\right]^2+\left[<|X(f)|\cdot|Y(f)|\cdot\sin(\Delta\varphi)>_\tau\right]^2}}{\sqrt{<|X(f)|^2>_\tau}\cdot\sqrt{<|Y(f)|^2>_\tau}}, \tag{3}$$

while the exponential of CCCF gives the relative phase function (RPF)

$$\alpha_{xy}(f)=\mathrm{Arg}\{<|X(f)|\cdot|Y(f)|\cdot\cos(\Delta\varphi)>_\tau+\mathbf{i}<|X(f)|\cdot|Y(f)|\cdot\sin(\Delta\varphi)>_\tau\}. \tag{4}$$

The estimator "$< >_\tau$" is computed using the moving average technique of size τ over the full length of the series. When amplitudes and phases are statistically independent equations (3-4) become:

$$\left|\gamma_{xy}(f)\right|=\frac{<|X(f)|\cdot|Y(f)|>_\tau}{\sqrt{<|X(f)|^2>_\tau}\cdot\sqrt{<|Y(f)|^2>_\tau}}\times\sqrt{\left[<\cos(\Delta\varphi)>_\tau\right]^2+\left[<\sin(\Delta\varphi)>_\tau\right]^2}, \tag{3′}$$

$$\alpha_{xy}(f)=\mathrm{Arg}\{<\cos(\Delta\varphi)>_\tau+\mathbf{i}<\sin(\Delta\varphi)>_\tau\}. \tag{4′}$$

The first factor in equation (3) is the amplitude coherence function (ACF)

$$\left|\gamma_{xy}(f)\right|_A=\frac{<|X(f)|\cdot|Y(f)|>_\tau}{\sqrt{<|X(f)|^2>_\tau}\cdot\sqrt{<|Y(f)|^2>_\tau}}, \tag{5}$$

while the second is the phase coherence function (PCF)

$$\left|\gamma_{xy}(f)\right|_P=\sqrt{\left[<\cos(\Delta\varphi(f))>_\tau\right]^2+\left[<\sin(\Delta\varphi(f))>_\tau\right]^2}. \tag{5′}$$

Phase synchronization has a twofold influence: directly on RPF according to equation (4'), and on CF via PCF according to equation (5'). A phase synchronized state is defined as a state in which the probability distribution function (PDF) is significantly different from a flat one (Rosenblum *et al.*, 2001). Suppose two extreme cases:

i) The case of perfect synchronization, i.e. the PDF of the phase difference is a Delta Dirac distribution:

$$\int \cos(\Delta\varphi)\cdot\delta(\Delta\varphi-\varphi_0)\cdot d(\Delta\varphi)=\cos\varphi_0, \qquad \int \sin(\Delta\varphi)\cdot\delta(\Delta\varphi-\varphi_0)\cdot d(\Delta\varphi)=\sin\varphi_0.$$

In this case, RPF given by equation (4′) has a constant value $\varphi_0(f)$:

$$\alpha_{xy}(f) = <\varphi_x(f)-\varphi_y(f)>_\tau = \int \Delta\varphi\cdot\delta(\Delta\varphi-\varphi_0)\cdot d(\Delta\varphi)=\varphi_0(f),$$

and PCF from equation (5′) is equal to unit at the investigated frequency $\left|\gamma_{xy}(f)\right|_P = 1$.

ii) The case of complete lack of synchronization, i.e. the phase differences are equally likely over the interval $(-\pi,\pi)$ and both integrals of the harmonic functions are zero:

$$\int \cos(\Delta\varphi)\cdot\frac{1}{2\pi}\cdot d(\Delta\varphi)=0, \qquad \int \sin(\Delta\varphi)\cdot\frac{1}{2\pi}\cdot d(\Delta\varphi)=0.$$

Consequently, RPF is zero:

$$\alpha_{xy}(f) = <\varphi_x(f)-\varphi_y(f)>_\tau = \int \frac{1}{2\pi}\cdot\Delta\varphi\cdot d(\Delta\varphi)=0,$$

also resulting in zero valued PCF $\left|\gamma_{xy}(f)\right|_P = 0$.

RPF is meaningful at frequencies where there is a significant PCF value. While ACF remains significant even in the case of unsynchronized, independent series, PCF discriminates between the cases with zero RPF with and without phase synchronization: in the first case, PCF is unity, while in the second it is zero. Particularly, PCF has the highest sensitivity to co-movements that affect equally both series in the couple like the bilateral import-export. For these reasons and different from the literature, in the present paper PCF was chosen as the indicator of phase synchronization. Since the time shift is investigated across a band of frequencies, it is useful to define the PCF index as a similarity measure of the bivariate process $x(t)$,$y(t)$ according to the Euclidean metric across the band:

$$\Gamma = \sqrt{\sum_f \left|\gamma_{xy}(f)\right|_P^2}. \tag{6}$$

Any computed time shift should be valid if the PCF index is greater than a threshold $\Gamma \geq \Gamma_{Th}$ to be established in Section 4.1. Equations (1-6) are implemented with the capabilities of Mathematica software using the modified formulae of Welford (Welford, 1962) and the observations on the computation algorithm (Scarlat and Mihăilescu, 2015).

II.2. Time Shift Based Connectivity

A connection between two coherent GDP series does exist when a statistical significant temporal shift is detected, provided that the PCF index is greater than the threshold.

RPF measures systematic time shifts (lags or advances, if any) between specified frequency bands of two series. If a band limited process appears in both series with a constant time lag, then RPF scales with frequency (Rosenberg *et al.*, 1989). Hereafter, the following notations of time parameters apply: the sampling step Δt_s=1Q, the band 2-4Q, the real time gap, δt, and the estimated time gap, $\delta\hat{t}$. Obviously, it is desirable the computed time gap is as close as possible to the real time gap, $\delta\hat{t} \to \delta t$. According to the time shift theorem, when artificially delaying the samples in the second series by one sampling step, $y(t)=x(t-\Delta t_s)$, the RPF given by equation (4′) is resulting in a linearly dependent phase shift across the band:

$$\alpha_{xy}(f) = 2\pi f \cdot \Delta t_s \cdot \tag{7}$$

Conversely, under some constraints, the RPF of any couple of series could be regressed over the investigated band and subsequently checked for the existence of a statistically significant linear dependence in coordinates $(f,\alpha_{x,y})$:

$$\alpha_{xy}(f) \propto \hat{b} \cdot f \, , \tag{8}$$

where: $\hat{b}$ is the estimated slope of the regression. If such a significant slope does exist, it is converted in a band limited time displacement $\delta\hat{t} \propto \hat{b}$. Equation (7) is used to calibrate equation (8), so that the temporal gaps can be computed using the formula:

$$\delta\hat{t} \propto -\frac{\tau}{2\pi} \cdot \hat{b} \cdot \tag{9}$$

This conclusion should be cautiously drawn because the reciprocal of the time shift theorem is not valid. For this reason, the statistical significance of the estimated slope $\hat{b}$ is analysed using the *p*-value and the R^2 coefficient with the variance table for the fitted model (ANOVA). While the *p*-value indicates the contribution of the individual terms in the fitting model, the coefficient of determination R^2 indicates the straightness of data fit. One should note the method has the particular power to reveal time shifts smaller than the sapling step, $|\delta\hat{t}| < \Delta t_s$, in the range $-2\text{Q} < \delta\hat{t} < 2\text{Q}$. Specific details are provided by Scarlat (2015, 2014).

To conclude, a connection between two series does exist when two conditions are fulfilled: i) the PCF index is greater than a threshold $\Gamma \geq \Gamma_{Th}$ as enabler, and ii) the RPF is scaling with frequency such that the estimated time shift is statistically significant.

II.3. Clusters

Cluster development or economic clustering based on distance measures was proposed to address the problem of countries' convergence from the viewpoints of business cycle synchronization and σ-convergence. Here we use a more restrictive approach to evaluate the connections based on the existence of the time gaps defined at the end of the previous section and to establish groups based on the above links. The number and the size of the groups depend on the binary decision on the existence of the gaps, which in turn is a function of the statistical significance imposed to equation (9).

An element is any economic entity exhibiting a GDP time series. The economic entity can be a country or an association of countries like the Eurozone-12. By analogy to the knot in an electric network, any element connected to at least three distinct elements gives rise to a cluster. Any element that belongs to a cluster is a member of the cluster. The member connected to all members of the cluster is the representative of the cluster. Since an element can belong to more than one cluster and any element could be counted several times, the cluster structure is of soft type and the clusters can overlap each other. One should note that it is not mandatory that all members of the cluster are connected to each other, but it is mandatory that they are connected with the representative. Therefore, the simplest cluster has four members (out of which one is the representative) and three links (the connections between the representative and each member). For every cluster, the local clustering coefficient (LCC) is defined as the ratio of the effective links between the members within the cluster except for the representative to the maximum number of links that could possibly exist among the members (Watts and Strogatz, 1998). The LCC measures to what extent the cluster achieves complete connectivity, i.e. all members in the cluster are connected to each other.

III. Data

Twenty four quarterly sampled GDP variations (percentage changes over the previous period), seasonally adjusted, were acquired from the Eurostat database (Eurostat, 2015) and correspond to the interval 1999Q1-2014Q4, i.e. 64 values. The countries that joined progressively the Eurozone were not considered. The codes of the countries correspond to ISO 3166-1 (ISO, 2015) are as follows: i) Twelve countries included in Eurozone-12 (EZ), namely Germany (DE), France (FR), the Nertherlands (NL), Austria (AT), Finland (FI), Belgium (BE), Italy (IT), Spain (ES), Luxembourg (LU), Portugal (PT), Ireland (IR), and Greece (GR); ii) The aggregated GDP of the whole group of EZ-12 consisting in the previously listed countries; iii) Nine European countries which preserve their own currency for the time being: the United Kingdom (UK), Sweden (SE), Denmark (DK), Poland (PO), the Czech Republic (CZ), Hungary (HU), Romania (RO), Norway (NO), Switzerland (CH); iv) the United States of America (US) and Japan (JP). The GDPs involved in the study are expressed in twelve distinct currencies.

The series were coupled in 276 mutual pairs and subsequently tested for the existence of the time shifts.

IV. Results and Discussions

IV.1 Enabling the Time Shifts with PCF

As stated in Section 2.2, any computed time gap is meaningful in the presence of a significant value of the PCF index. The lower threshold of the PCF index defined by equation (5′) is established to be statistically significant with respect to the null hypothesis of hazardous appearance. Because of the shortness of the series, the method of surrogates was preferred here (Sprott, 2003). In every GDP pair, one of the series was shuffled 1000 times and the corresponding PCF indices were averaged both over the number of shuffles and over the number of possible couples. Table 1 shows the mean values of the PCF index, the ACF index, defined similarly to equation (6), and the corresponding errors of the means as well, including the synthesized white Gaussian series as benchmark. The ACF given by equation (5) is little influenced by shuffling, thus confirming our choice that PCF is a more appropriate indicator for phase synchronization.

Table 1

PCF and ACF Indices in the Cases of GDP Shuffles and White Gaussians

	Mean		Standard deviation σ	
	PCF index Γ	ACF index	PCF index	ACF index
GDP surrogates	0.137	0.784	0.027	0.016
White Gaussian	0.119	0.794	0.021	0.020

The surrogates impose higher constraints than the Gaussians; therefore, they account for the lower threshold in the limit of 3σ:

$$\Gamma \geq \Gamma_{Th} = 0.219. \tag{10}$$

To conclude, any PCF index greater than the threshold stated by equation (10) enables further estimation of a possible time shift.

IV.2 Connectivity and Clusters

190 distinct pairs out of the total number of 276 investigated couples qualify according to equation (10). Initially, the quantities R^2 and p-value were set to the initial values:

$$p\text{-value} \leq 5\%, \quad R^2 \geq 0.75. \tag{11}$$

Only 24 connections were found among 14 distinct elements grouped in eight overlapping clusters, with 51 members (the elements are multiply-counted) using nine different currencies. The numbers of connections by currency among national GDPs are summarized in Table 2. Apart from the most traded European currencies, the EUR, the GBP, and the CHF, the other currencies are symbolized by triple X or triple Y; the figures when the aggregate EZ has been dropped off are in brackets.

Table 2

The Statistics of Connections among GDPs by Currency

Type of connections→ No. of connections↓	EUR-EUR	EUR-XXX	XXX-YYY	Total
Possible	78 (66)	143 (132)	55 (55)	276 (253)
Effectively found	5 (2)	13 (11)	6 (6)	24 (19)

The findings suggest there are more connections between GDPs expressed in different currencies than in the same currency, the EUR. The same method applied to the exchange rates with respect to the USD did not reveal similar connections in the specified 2-4Q band. Therefore, the results are not the consequence of the exchange rates dynamics, but more likely of the effects of trades and their subsequent financial flows between countries, because the bilateral import-export activities operate synchronously onto the variations of each series in the pair resulting in a theoretical high PCF index. Time gaps computed according to equation (9) are given in Table 3. The standard errors of the means are included in the table. Due to the antisymmetry of RPF, the time shifts between representatives in different clusters appear twice, with opposite signs.

The facts reveal synchronized, time shifted packages of cycles in the range 2-4Q embedded in the GDP variations. One should note our results are different from the classic dendrograms, firstly because the traditional clustering methods are focused on the co-movements of business cycles, typically in the range 6-32Q, i.e. outside our range of investigation, and secondly, because they use exclusively coherence measures derived from PCP, ACF, or CF, and not time shifts. However, strong DE-UK, DE-FR and FR-UK ties were commonly found in literature, irrespective of the clustering techniques (Hampel *et al*., 2015).

From the time shift perspective, the biggest clusters were found around CH (ten members, nine links), followed by UK and FI (eight members and seven links each), DE and EZ (six members and five links each), then FR (five members, four links), RO and HU (four members and three links each). Note that the greatest clusters emerge around the representatives that use the most traded European currencies: the EUR, the GBP, and the CHF; only the representatives of the smallest clusters of RO and HU are using other currencies. Since data is mostly from the European area, the USD and the JPY currencies do not appear as important.

The DE and EZ groups have the same elements and the greatest LCCs (second column in Table 2). The LCCs diminish when more currencies are involved in the clusters: two distinct currencies in the EZ or DE clusters, and seven in the CH cluster. Following Watts and Strogatz (1998), the EZ and DE clusters are close to fully connected "small-worlds" under the influence of the common currency, the EUR. From technical perspective, the aggregate EZ-12 is hiding intra-EZ trades of a particular member state of EZ with the aggregate itself, because the import-export figures of the member state are vanishing in the aggregate GDP. The consequence is that the EZ cluster should contain two kinds of members: the members of EZ-12 with significant trade outside EZ-12, and the nonmembers of EZ-12 with significant trade with EZ-12, like UK, CH, and even RO and HU. Focusing on the connections of the aggregate EZ-12, the persistent connections EZ-DE and EZ-FR make sense due to the important weights of the German and French trades outside the Eurozone-12. Yet, there is no satisfactory explanation for the case of EZ-FI.

Table 3

Clusters of GDP Series

Cluster reprez.	Local clustering coefficient	Connections	PCF index Γ	Slope $\hat{b}$	Std. error	p-value (%)	R^2	Estimated $\delta\hat{t}$ (days)
CH	0.19	CH - UK	0.538	–0.587	0.057	0.20	0.972	102±10
		CH - FI	0.715	–0.298	0.040	0.49	0.933	52±7
		CH - RO	0.620	–0.634	0.094	0.67	0.938	110±16
		CH - IT	0.591	–0.810	0.160	1.49	0.895	141±28
		CH - EZ	0.718	–0.604	0.125	1.68	0.887	105±22
		CH - BE	0.728	–0.291	0.067	2.21	0.865	51±12
		CH - DE	0.732	–0.611	0.145	2.46	0.855	106±25
		CH - JP	0.660	–0.642	0.195	4.59	0.783	94±34
		CH - SE	0.515	–0.361	0.111	4.76	0.779	63±19
UK	0.43	UK - DE	0.775	0.128	0.012	0.17	0.975	–22±2
		UK - CH	0.538	0.587	0.057	0.20	0.972	–102±10
		UK - EZ	0.807	0.108	0.013	0.35	0.959	–10±2
		UK - HU	0.620	0.259	0.035	0.53	0.947	–45±6
		UK - PO	0.284	0.573	0.085	0.67	0.938	–100±14
		UK - FR	0.573	–0.385	0.067	1.03	0.918	67±12
		UK - FI	0.570	–0.276	0.096	4.99	0.756	–48±16
DE	0.80	DE - FR	0.691	–0.516	0.044	0.13	0.979	90±8
		DE - UK	0.775	–0.128	0.012	0.17	0.975	22±2
		DE - EZ	0.973	–0.013	0.010	0.37	0.970	3±2
		DE - CH	0.732	0.611	0.145	2.46	0.855	–106±25
		DE - FI	0.776	0.166	0.048	4.15	0.797	–29±8
EZ	0.80	EZ - FR	0.762	–0.453	0.039	0.14	0.978	79±7
		EZ - UK	0.807	–0.108	0.013	0.35	0.959	10±2
		EZ - DE	0.973	0.013	0.010	0.37	0.970	–3±2
		EZ - CH	0.718	0.604	0.125	1.68	0.887	–105±22
		EZ - FI	0.748	0.174	0.038	1.95	0.875	–30±7
FI	0.33	FI - CH	0.715	0.298	0.040	0.49	0.933	–52±7
		FI - RO	0.781	–0.209	0.033	0.83	0.929	36±6
		FI - HU	0.489	0.612	0.106	1.04	0.917	–106±18
		FI - EZ	0.748	–0.174	0.038	1.95	0.875	30±7
		FI - DE	0.776	–0.166	0.048	4.15	0.797	29±8
		FI - US	0.602	0.789	0.278	4.98	0.758	–137±48
		FI - UK	0.570	0.276	0.096	4.99	0.756	48±16
FR	0.67	FR - DE	0.691	0.516	0.044	0.13	0.979	–90±8
		FR - EZ	0.762	0.453	0.039	0.14	0.978	–79±7
		FR - UK	0.573	0.385	0.067	1.03	0.918	–67±12
		FR - HU	0.553	0.624	0.166	3.30	0.825	–108±29
RO	0.67	RO - CH	0.620	0.634	0.094	0.67	0.938	–110±16
		RO - FI	0.781	0.209	0.033	0.83	0.929	–36±6
		RO - BE	0.664	0.116	0.038	4.90	0.760	–20±7
HU	0.67	HU - UK	0.620	–0.259	0.035	0.53	0.947	45±6
		HU - FI	0.489	–0.612	0.106	1.04	0.917	106±18
		HU - FR	0.553	–0.624	0.166	3.30	0.825	108±29

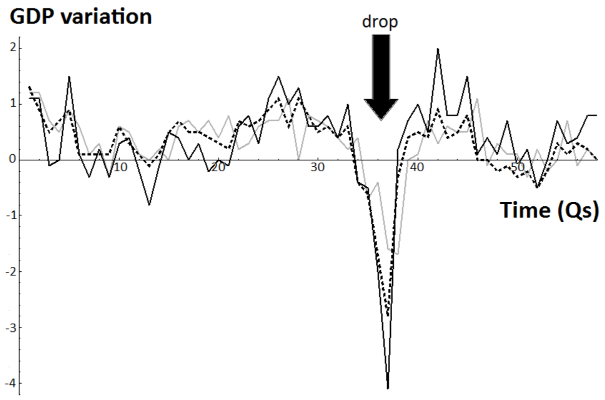
A special attention shall be given to the extreme timing of EZ-DE connection. The corresponding GDP variation exhibits the greatest PCF index and the smallest time shift detected in the present work, of around three days. It is the particular case of almost perfect synchronization with PCF almost unit and RPF almost zero. This result seems to be a weird one, because the German GDP is accounting for less than one third of the aggregated GDP of the Eurozone. Moreover, as the second economy of the Eurozone, the French GDP exhibits a significant time delay of around one quarter with respect to both DE and EZ; therefore, it is not the volume of GDP that causes the approximate zero-lag in the couple DE-EZ. The timing mechanism Germany seems to impose on the Eurozone is rather related to changes than to levels: a sharp change in the German GDP is fully recognizable in the variation of the aggregated GDP of the Eurozone (see Figure 1), while the variations of the GDPs of the other member states are averaging out. The explanation should be of more intricate nature and should be looked for in the European stock markets. According to Călin (2015), who used the MIDAS methodology to solve the problem of different frequencies, the risk in the investments in Europe is mainly driven by the German macroeconomic dynamics via its stock markets.

Neglecting the aggregate EZ-12, the connections seem to be not effective when the bilateral trades use the same currency. The exception DE-FR – with greatest statistical significance among all estimates of the time gaps – works for the largest trading volumes inside EZ and for the biggest correlation of the volatility dynamics between the German and French stock markets (Călin, 2015). Another exception is DE-FI, which may be seen as a consequence of the identity between the clusters of DE and EZ. However, this issue needs deeper analysis, since it seems not to agree with the postulate of “endogeneity of the optimum currency area criteria” (Frankel and Rose, 1998); we restrict to this the facts mentioned previously.

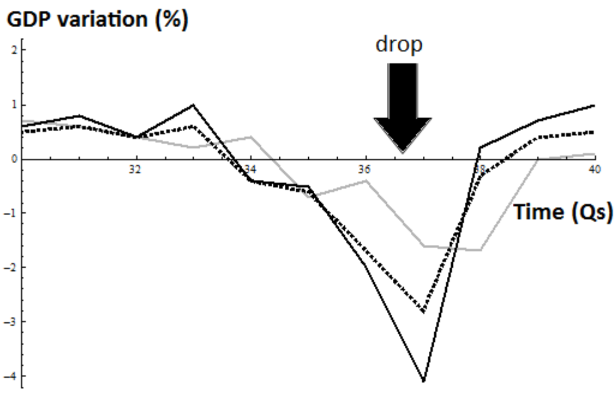
Since it is commonly known that the short cycles are generated by time spikes like fluctuations or shocks, the time shift based connectivity might be suitable to reveal the exogenous channels along which such shocks propagate. According to Fornari and Stracca (2013), this matters not only in crises but also in normal times, irrespective of the financial development or the financial structure of a given country.

Figure 1

GDP Drop of DE and the Subsequent Drops of FR and EZ



a) GDP variations of DE (black), EZ (gray), and FR (dashed)



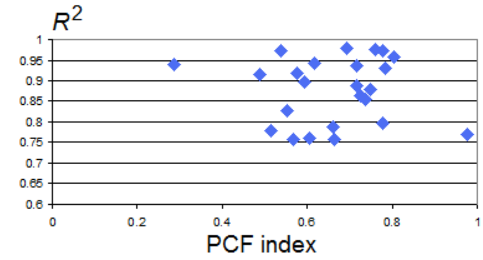
b) Detail of GDP variations of DE (black), EZ (gray), and FR (dashed)

IV.3. The Time Map

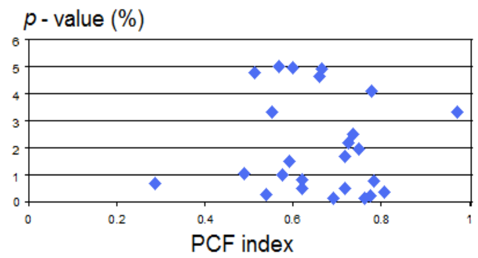
The facts show that the PCF index is not correlated with the R^2 coefficient (Figure 2a) or with the *p*-values (Figure 2b). On the other side, *p*-value and R^2 are negatively correlated: the closer to zero the *p*-value, the closer to one R^2, and the greater the meaningfulness of the subsequent time gap given by equation (9) with the disadvantage of reducing the number of qualified candidates (Figure 2c). The diminishing number of distinct connections is indicated in Table 3 using shadowed rows: dark grey for 1%<*p*-value≤5%, light grey for 0.5%<*p*-value≤1%, and no shadow for *p*-value≤0.5%. The dependence on the *p*-value of the number of connections and clusters is indicated in Figure 2d in double vertical scales.

Figure 2

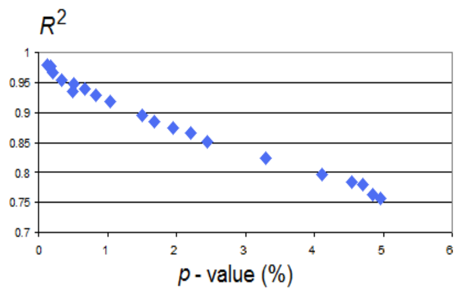
R^2 Coefficient and *p*-value vs. PCF Index



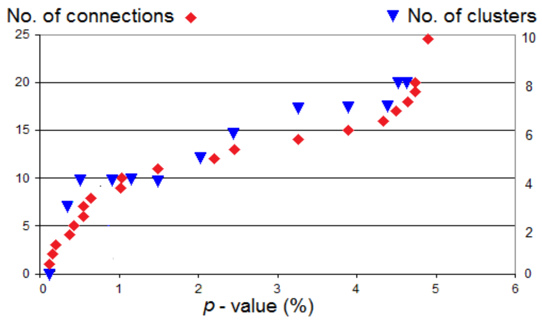
a) R^2 and PCF index are not correlated



b) p-value and PCF index are not correlated



c) R^2 and p-value are negatively correlated



d) Number of connections and clusters vs. p-value

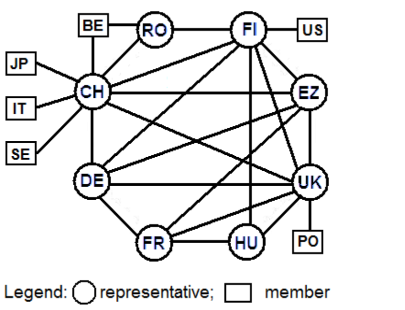
The initial connectivity network is shown in Figure 3a; when imposing the more restrictive constraint

$$p\text{-value} \leq 0.5\%, \tag{11'}$$

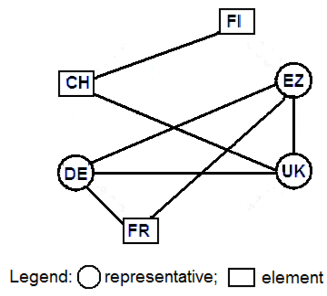
the network reduces down to the most persistent connections among three residual clusters of EZ, DE and UK, and three separate elements (Figure 3b).

Figure 3

Connectivity Networks



a) Network connections among initial clusters

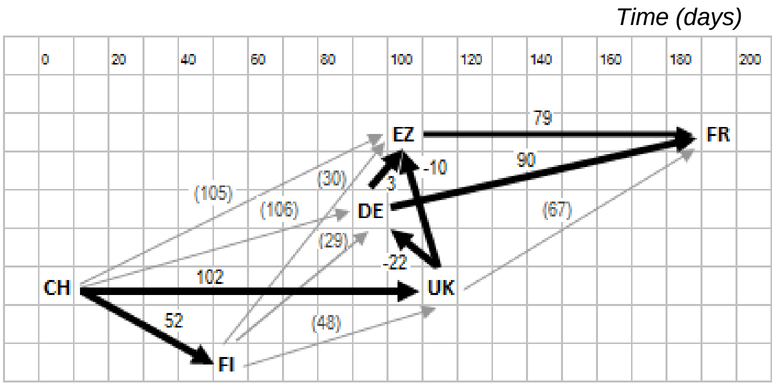


b) Network connections at lower p-values

The temporal sequence of the reduced network shown in Figure 3b is indicated in Figure 4. The horizontal temporal scale of the time map is rounded to a resolution of 10 days. For obvious reasons, the time gaps were reshaped such that all shifts appear as delays with respect to the most advanced one.

Figure 4

Time Map



Time shifts among the elements of the network at lower p-values

Since the technique of computing the time shifts fails to preserve the property of additivity, it is possible to have different estimations of the gaps between the same elements. As a general rule, the reliable value is the one where the time gap is directly evaluated with respect to the reference. The shortcoming is reduced when improving the statistical significance by replacing equation (11) with the more restrictive constraints of equation (11'). The results are consistent between the limits of the standard errors as demonstrated by comparing the time shifts among the remaining elements shown in Figure 3b (as indicated in Figure 4 in the vicinities of the black thick arrows) with the appropriate shifts with respect to the representatives in the initial clusters presented in Figure 3a (in round brackets near the gray thin arrows in Figure 4). The arrows are not necessarily indicating the real traces of the shocks. A more realistic interpretation is that the map is revealing different propagation times diverging from the US, which is located before all elements: it foregoes 135 days FI in the unique cluster wherewith connected, so that a shock needs roughly 200 days or 2.2Q to propagate from the US to the Eurozone. This is indeed the lower limit of the time span mentioned in the early stages of the analyses related to the financial crises (IMF, 2009) and confirmed in post-crisis.

V. Conclusions

This paper reveals the connectivity among quarterly GDP series collected at the same times by identifying time shifts between cycles in the 2-4Q band that contribute at most to point-to-point variations of GDPs. The method relies on the theory of the complex-valued cross coherence function particularly on the scaling of the relative phase function across the band provided that there is a minimum value of the phase coherence index. The connectivity based on time shifts is superior to other connectivity relationships since it combines two independent features, namely the coherence, which measures the statistical presence of a common component in both series at a certain frequency, and the scaling of the relative phase across the band, so that knowing the sense and the magnitude of the movements in the advanced series might indicate the trends in the delayed one. Apart from the particular assessment of the connectivity, the technique allows to detect time shifts smaller than the sampling period of the series instead of integer multiples of the sampling step as usual. The method is highly sensitive to co-movements that affect equally both series in the couple, such as the bilateral trades or the financial flows going along with the trades.

Fourteen economic systems out of twenty-four investigated were grouped into eight clusters whose GDP series are connected in terms of time advances or lags with respect to the representative of the cluster. Due to the focus on the European area, the largest clusters were found around the representatives that use the most traded European currencies: CHF, GBP, and EUR. Connections exist mainly between GDPs expressed in different currencies. Since no similar connections were found in the associated exchange rates, it supports the idea that the connections are likely to be the direct consequence of the trades and their corresponding financial flows.

EZ and DE exhibit negligible time delay between them. Moreover, the DE and EZ clusters are very similar in terms of elements and clustering coefficient. For these reasons, the German GDP is considered the driving force of the changes in the GDP of the Eurozone-12.

Deepening the analysis of statistical significance, the residuals of the largest clusters were chronologically mapped to reveal different propagation times of the fluctuations originating probably from the US, whose variations were preceding all the elements involved in the present study. There are no quantitative estimations of such time delays in literature.

The method may be useful for calculating speeds of contagion, spillover effects, or further breakdowning of the sampling period in smaller intervals to lay the input variables in correct time positions in the case of mixed frequency models or in the case of multivariate correlations of ARMA type.

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