5. FORECASTING THE GOVERNMENT YIELD CURVE IN CHINA: A CYCLICAL REVERTING MEAN APPROACH

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Abstract

In this paper, we allow the Chinese interest rate to move cyclically and introduce an extension of Vasicek (1977) model to estimate Chinese yield curve in response to the cyclical movements of interest rates. In this model, the constant long-run reverting mean is replaced by a Fourier series to capture the cyclical behaviour of instantaneous rates. We use the daily inter-bank zero-coupon yields data ranging from 2006 to 2015. The extension model is found to perform significantly better than the benchmark in both in-sample fitting and out-of-sample forecasting.

Keywords: term structure of interest rates; Fourier series; cyclical movement; interest rates forecasting; Chinese yields curve

JEL Classification: C31 E32 E43 F37

1. Introduction

One of the fundamental features of interest rates is the cyclical behaviour which was discussed in Kessel (1971), Friedman (1986), Roma and Torous (1997), among others. According to the literature of advanced economies, the specific interest rates cycle is related to the business cycle with increasing of interest rates at business expansions and decreasing at contractions. We are interested in examining whether the assumption of the cyclical tendency of interest rate could help to describe the whole yield curve in China. Several studies explored the Chinese term structure of interest rates based on one-factor short-rate models. The Vasicek (1977) model is found to provide good in-sample fitting to Chinese yield curve (Xie and Wu, 2002 and Lin and Zheng, 2005).

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Therefore, we allow the Chinese interest rates to move cyclically and introduce an extension of Vasicek model, which integrates the cyclical effect of interest rates, to model Chinese term structure of interest rates. Following Moreno, Novales and Platania (2018), the constant long-run equilibrium level in the Vasicek model is replaced by a time-varying Fourier series to capture the cyclical behaviour of interest rates. We are interested in the question whether this model could provide a better estimation of Chinese yield curve when the Fourier series is incorporated and to what extent. To the best of our knowledge, this study is the first to bring in the cyclical effect to the estimation and prediction of Chinese term structure of interest rates.

This paper is organized as follows. Section 2 presents methodology. Section 3 provides the data description and empirical analysis. The last section concludes.

2. Methodology

The Model

In the Vasicek model, the instantaneous interest rate is assumed to converge to a long run equilibrium constant value, while in this cyclical mean reversion model, the constant value is replaced by a cyclical long-term level described as a Fourier series. The Fourier model specifies that the instantaneous interest rate denoted by r_t follows the Ornstein-Uhlenbeck process which is expressed by the stochastic differential equation as below:

$$dr_t = \kappa(f(t) - r_t)dt + \sigma dW_t \tag{2.1}$$

where: $\kappa, \sigma \in \mathbb{R}^+$ and W_t is a standard Wiener process. The cyclical mean reversion level denoted by $f(t)$ is assumed to follow a Fourier series as:

$$f(t) = \sum_{n=0}^{\infty} Re[A_n e^{in\omega t}] \tag{2.2}$$

where: $\forall n \mid A_n \in \mathbb{C}$. The phase factor contained in A_n could be defined as $A_n = A_{n,x} + iA_{n,y}$ where: $A_{n,x}, A_{n,y} \in \mathbb{R}$. $A_{n,x}$ is the amplitude of the instantaneous rate fluctuations and the $A_{n,y}$ is the phase. Since only the real part of the Fourier series has an economic meaning, only real part is considered.

Under the risk-neutral measure $\widetilde{P}$, the standard Wiener process is expressed as $\widetilde{W_t} = W_t + \lambda t$, where the market price of risk $\Lambda(r_t, t)$ is a constant which equals to λ . Then the risk-neutral vision of the SDE in (2.1) could be given as below:

$$dr_t = \mu_r dt + \sigma d\widetilde{W_t} \tag{2.3}$$

where:

$$\mu_r = \kappa(\alpha + g(t) - r_t) \tag{2.4}$$

$$\alpha = A_0 - \frac{\lambda\sigma}{\kappa} \tag{2.5}$$

$$g(t) = \sum_{n=1}^{\infty} Re[A_n e^{in\omega t}] = f(t) - A_0 \tag{2.6}$$

By using the Itō's lemma, no-arbitrage constraint and probabilistic techniques, the price of a zero-coupon bond at time t with maturity *T* and par value £1 is expressed as

$$P(r_t, t, T) = e^{A(t,T) - B(t,T) r_t} \tag{2.7}$$

where:

$$A(t,T) = \frac{\sigma^2}{2\kappa^2}\left[(T-t) - 2B(t,T) + \frac{1-e^{-2\kappa(T-t)}}{2\kappa}\right] + \big(B(t,T) - (T-t)\big)\alpha - \sum_{n=1}^{\infty} Re[\frac{A_n}{n\omega(\kappa+in\omega)}(e^{in\omega t}\big(n\omega e^{-\kappa(T-t)} + i\kappa - n\omega\big) - i\kappa e^{in\omega T})] \quad (2.8)$$

$$B(t,T) = \frac{1-e^{-\kappa(T-t)}}{\kappa} \quad (2.9)$$

Since the yield to maturity $R(r_t,t,T)$ could be given in form of bond price $P(r_t,t\,T)$ as follows,

$$R(r_t,t,T) = -\frac{1}{\tau} lnP(r_t,t\,T), \tau = T-t \quad (2.10)$$

we plug in the expression of bond price in (2.7) and keep only the first term of the Fourier series for simplicity, then the in-sample fitting model could be given for each maturity j as

$$Y_{j,t} = \delta_1 z_{1j,t} + \delta_2 z_{2j,t} + \delta_3 z_{3j,t} + \delta_4 z_{4j,t} + u_{j,t} \quad (2.11)$$

where:

$$Y_{j,t} = R(r_t,t,T) - \frac{B(t,T)}{T-t} r_t$$

$$z_{1j,t} = \frac{B(t,T)}{T-t} - 1$$

$$z_{2j,t} = \frac{1}{2\kappa^2} - \frac{B(t,T)}{(T-t)\kappa^2} + \frac{1-e^{-2\kappa(T-t)}}{4(T-t)\kappa^3}$$

with $B(t,T) = \frac{1-e^{-\kappa(T-t)}}{\kappa}$, $\delta_1 = \alpha$, $\delta_2 = \sigma^2$, $\delta_3 = A_x$ and $\delta_4 = A_y$, $u_{j,t}$ is the error term.

The first term of the Fourier series we have taken could be given in this form:

$$Re[-(A_x + iA_y)(\frac{e^{i\omega t}(\omega e^{-\kappa(T-t)} + i\kappa - \omega) - i\kappa e^{i\omega T}}{\omega(\kappa+i\omega)})] \quad (2.12)$$

where: $A_x + iA_y = A_1$. Then, by using the Euler's formula $e^{it} = cost + isint$, (2.12) could be rewritten as

$$\frac{A_x}{\omega(\kappa^2+\omega^2)(T-t)}\{-\kappa\omega\cos(\omega t)\,e^{-\kappa(T-t)} - \kappa^2(\sin(\omega T) - \sin(\omega t)) - \omega^2\sin(\omega t)\left(e^{-\kappa(T-t)} - 1\right) + \kappa\omega\cos(\omega T) +$$

$$\frac{A_y}{\omega(\kappa^2+\omega^2)(T-t)}\{\kappa\omega\sin(\omega t)\,e^{-\kappa(T-t)} - \kappa^2(\cos(\omega T) - \cos(\omega t)) - \omega^2\cos(\omega t)\left(e^{-\kappa(T-t)} - 1\right) - \kappa\omega\sin(\omega T)$$

$$= A_x z_{3j,t} + A_y z_{4j,t} \quad (2.13)$$

with

$$z_{3j,t} = \frac{1}{\omega(\kappa^2+\omega^2)(T-t)}\{-\kappa\omega\cos(\omega t)\,e^{-\kappa(T-t)} - \kappa^2(\sin(\omega T) - \sin(\omega t)) - \omega^2\sin(\omega t)\left(e^{-\kappa(T-t)} - 1\right) + \kappa\omega\cos(\omega T)\} \quad (2.14)$$

$$z_{4j,t} = \frac{1}{\omega(\kappa^2+\omega^2)(T-t)}\{\kappa\omega\sin(\omega t)\, e^{-\kappa(T-t)} - \kappa^2(\cos(\omega T) - \cos(\omega t)) - \omega^2\cos(\omega t)\left(e^{-\kappa(T-t)} - 1\right) - \kappa\omega\sin(\omega T)\} \quad (2.15)$$

2.2 Estimation Method

We wrote the model in form of a regression model as given in equation (2.11). However, this model could not be estimated by regression since the explanatory variables are dependent on the structural parameters κ and ω. As to this nonlinear optimization problem, following Moreno, Novales & Platania (2018), we estimate the model day by day. We use the everyday cross-sectional data of the interest rates to estimate the parameters $\kappa, \omega, \delta_1, \delta_2, \delta_3, \delta_4$ for each day, within the values of those, the minimum values of the sum of squared residuals in (2.11) are achieved. The sum of squared residuals in equation (2.11) is given as below:

$$SR(\hat{\theta}_t) = \sum_{j,t}[Y_{j,t} - (\delta_1 z_{1j,t} + \delta_2 z_{2j,t} + \delta_3 z_{3j,t} + \delta_4 z_{4j,t})]^2 \quad (2.19)$$

After the day-by-day estimation for each day, the time series of $\alpha, \sigma^2, A_x, A_y, \kappa$ and ω could be generated respectively and we express the five time-series as a structural parameter denoted by $\theta = (\alpha, \sigma^2, A_x, A_y, \kappa, \omega)$. Since the Vasicek model is nested in the Fourier model by setting $z_{3j,t} = z_{4j,t} = 0$ in (4.11), the same estimation method is applied to the Vasicek model and the structural parameters $\theta = (\alpha, \sigma^2, \kappa)$ are estimated.

2.3 Prediction Approach

Prediction is conducted by using a first-order autoregression. The parameters in the Fourier model are assumed to follow a first-order autoregressive process as a vector denoted by $\hat{\theta}$,

$$\hat{\theta} = \hat{c} + \gamma\hat{\theta}_{t-1} + \varepsilon_t \quad (2.20)$$

where: ε_t is white noise. As the Fourier model is based on a single factor, the instantaneous rate, we construct the prediction of it by using Euler discretization,

$$E[r_{t+\Delta t}|r_t] = r_t + \kappa(\mu - r_t)\Delta t \quad (2.21)$$

In this equation, Δt denotes the required forecast horizons set at 1, 5, and 21 in responding to one-day, one-week and one-month ahead forecasting. Also, μ is a nonlinear function of the structural parameters in the Fourier model while it is a constant parameter in the Vasicek model. With the prediction of parameters and the instantaneous rate, the interest rates with the other maturities $R(r_t, t, T)$, could be obtained by using equation (2.11).

3. Empirical Analysis

3.1. Data Description

Chinese inter-bank Zero-coupon yields is used with 2335 daily observations for each maturity from March 1st, 2006 to June 30th, 2015. Yields maturities included are 1, 3, 6 months and 1, 2, 3, 5, 7, 10, 20 and 30 years. The yields are annualized and given in percentage and the data source is ChinaBond[4].

The summary statistics of Chinese zero-coupon yields are reported in Table 1. The mean value increases gradually as the maturity moves longer. This result indicates that the

[4] *The yield curve is constructed by using bootstrapping on the coupon bonds in inter-bank market and Hermite interpolation is applied to smooth the yields as stated in ChinaBond.*

Chinese treasury yield curve is upward sloping. As given in the third column, the value of standard deviation has a decreasing trend with maturity which illustrates that the short-term yields are much more volatile than the long-term yields. According to the distribution statistics, the results show a lack of symmetry and a flatter distribution than Gaussian in the data. The value of skewness indicates the asymmetry from normal distribution, with positively skewed yields at maturities of one month and longer than 5 years, and negatively skewed yields at maturities from 3-month to 3-year. In addition, Chinese yields prove high level of persistence at all maturities.

Table 1. Summary Statistics of Chinese Zero-coupon Yields

Month	Mean	Std. Dev.	Kurtosis	Skewness	Min.	Max.	$\hat{\rho}(5)$	$\hat{\rho}(30)$
1	2.3140	0.0194	0.6263	0.6105	0.7102	6.5750	0.988	0.807
3	2.4896	0.0179	-0.6754	-0.1324	0.7989	5.1132	0.997	0.874
6	2.5479	0.0172	-0.8496	-0.2647	0.8183	4.3744	0.998	0.886
12	2.6440	0.0168	-0.8635	-0.3232	0.8871	4.2503	0.998	0.892
24	2.8554	0.0160	-0.7770	-0.2917	1.0700	4.4190	0.999	0.906
36	3.0281	0.0142	-0.6813	-0.2024	1.2437	4.5003	0.998	0.891
60	3.2887	0.0120	-0.8364	0.0180	1.7342	4.5293	0.998	0.870
84	3.4962	0.0109	-0.7953	0.0580	2.1223	4.6698	0.998	0.863
120	3.6753	0.0096	-0.8642	0.2937	2.6711	4.7222	0.997	0.855
240	4.1204	0.0080	-0.2348	0.4091	3.3782	5.0968	0.998	0.831
360	4.2349	0.0079	-0.3064	0.3665	3.4800	5.1988	0.997	0.834

3.2 In-sample Fitting of Yield Curve

To explore the level effects of the financial crisis in 2008 on Chinese government bond market, the yield curve will be fitted within the whole sample period from 03/01/2006 to 06/30/2015 and post-crisis period from 04/01/2009 to 06/30/2015, respectively.

Table 2 displays the parameters estimation of both periods. The parameter mean and standard deviation are given for both models. The minimized numerical value of the objective function and the sum of the absolute value of the pricing errors across all maturities are given in the last two rows to measure the fitting ability to data. In both sample periods, the Fourier models show better in-sample fitting performances than the Vasicek models with significantly lower values of both minimized objective function and the aggregate sum of squared errors. The Fourier model cut down 55.5% of the aggregate sum of squared errors both in the whole sample period and the post-crisis period. Furthermore, the aggregate sum of squared errors is reduced by 23.3% in the post-crisis period for both Fourier and Vasicek models than in the whole period. The better in-sample performance of both models over post-crisis period indicates significant impact of the global financial crisis on the Chinese yield curve.

Table 2. Parameter Estimation from In-sample Fitting

	2006 - 2015		2009 - 2015	
Parameter	Fourier	Vasicek	Fourier	Vasicek
$\hat{\delta}_1$	0.0477	0.0472	0.0474	0.0474
	(0.0001)	(0.0005)	(0.0002)	(0.0008)
$\hat{\delta}_2$	0.0004	0.0014	0.0002	0.0007
	(0.0000)	(0.0000)	(0.0000)	(0.0001)
$\hat{\delta}_3$	0.0018		0.0005	
	(0.0023)		(0.0126)	
$\hat{\delta}_4$	-0.0079		0.0026	
	(0.0022)		(0.0111)	
$\hat{\kappa}$	0.2435	0.3373	0.2290	0.2743
	(0.0010)	(0.0166)	(0.0014)	(0.0227)
$\hat{\omega}$	4.9044		5.2498	
	(0.0232)		(0.0310)	
$\sum_{i,t} \min SR(\hat{\theta}_{i,t})$	0.0244	0.1051	0.0198	0.0861
$\sum_{i,t} \lvert\hat{u}_{i,t}\rvert$	13.8489	31.1426	10.6244	23.8960

Note: The estimation standard errors are indicated in parentheses

The goodness of fit statistics of yields with each maturity is given in Table 3 for both models within two sample periods. Both Fourier and Vasicek models fit the yield curve well with low pricing errors at each maturity. However, the Fourier model outperforms the Vasicek with lower numerical pricing errors at all maturities. For example, the sum of squared errors lies below 0.006 in Fourier model while in the Vasicek model all the sum of squared errors at different maturity are less than 2. Interestingly, both models provide much better approximation of the data within the post-crisis sample period than in the whole period. The pricing errors over all the maturities are relatively lower by using the post-crisis period than the whole period for both Fourier and Vasicek models except for the one-year error in Fourier. Thus, the goodness of fit result illustrates that the Chinese yield curve is very sensitive to the global financial crisis in 2018.

Table 3. Goodness of Fit of Yields with Each Maturity

	2006 – 2015					2009 - 2015			
Maturity	Fourier		Vasicek			Fourier		Vasicek	
	$\sum_t \hat{u}_t^{\,2}$	$\sum_t \lvert\hat{u}_t\rvert$	$\sum_t \hat{u}_t^{\,2}$	$\sum_t \lvert\hat{u}_t\rvert$		$\sum_t \hat{u}_t^{\,2}$	$\sum_t \lvert\hat{u}_t\rvert$	$\sum_t \hat{u}_t^{\,2}$	$\sum_t \lvert\hat{u}_t\rvert$
3M	0.0055	1.9684	0.0202	4.6675		0.0038	1.4620	0.0142	3.3448
6M	0.0031	1.3633	0.0215	4.8014		0.0021	1.0246	0.0171	3.6527
1Y	0.0031	1.3886	0.0182	4.1063		0.0059	1.1421	0.0163	3.4461
2Y	0.0031	1.5893	0.0117	3.2030		0.0023	1.1190	0.0108	2.7737
3Y	0.0027	1.4361	0.0091	2.7131		0.0023	1.1240	0.0079	2.0772
5Y	0.0016	1.2191	0.0068	2.7412		0.0016	0.9839	0.0058	2.0598
7Y	0.0010	1.0993	0.0043	2.1752		0.0008	0.7567	0.0035	1.5283
10Y	0.0015	1.4564	0.0035	2.0436		0.0013	1.1724	0.0031	1.5927
20Y	0.0012	1.1681	0.0035	2.0569		0.0010	0.9247	0.0027	1.4959
30Y	0.0017	1.1599	0.0062	2.6345		0.0016	0.9150	0.0047	1.9248

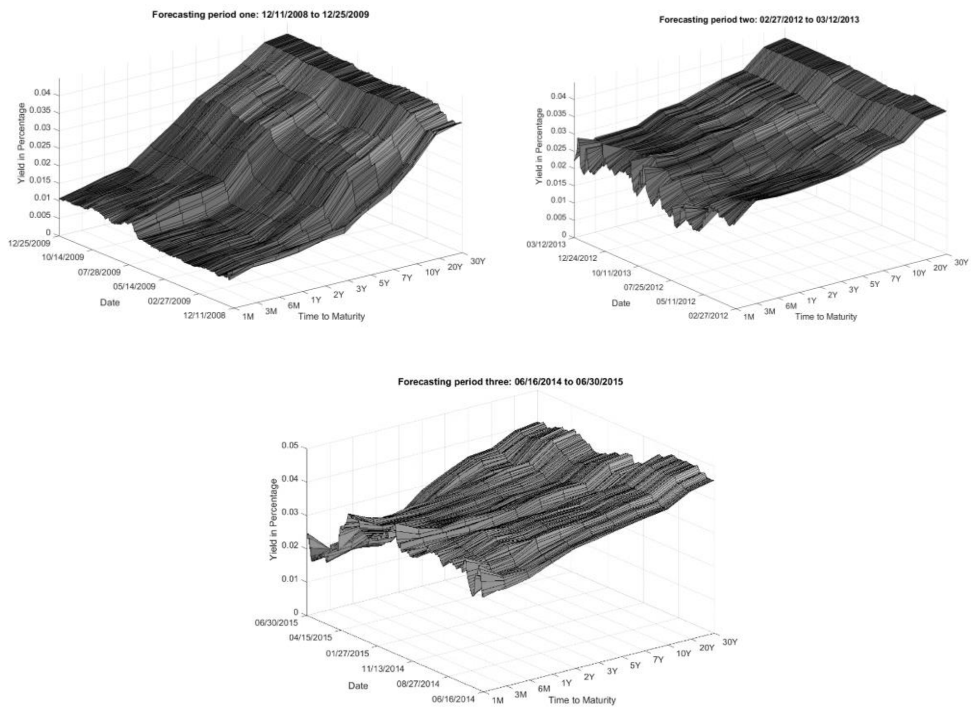
3.3 Out-of-Sample Forecasting

To explore the forecasting power of the Fourier model in a different situation, we choose three time periods with various shapes of yield curve such as the forecasting periods which cover the slots 12/11/2008 to 12/25/2009, 02/27/2012 to 03/12/2013 and 06/16/2014 to 06/30/2015. Each forecasting period contains 260 trading days and roughly covers one year. Both Vasicek and Fourier models are estimated from 03/01/2006 to the day each forecasting period started and forecast the value of each parameter within the forecasting horizons of one-day, one-week and one-month respectively.

During the first forecasting period from Dec. 2008 to Dec. 2009, the Chinese economy touched the bottom and began to show signs of recovery from the global financial crisis. As displayed in Figure 1, the yield curve is steeply upward sloping with very low rates at the short- end. In addition, approximately from Sep. 2009, the level of yield curve at all the maturities moved up slightly which might be an indication of full recovery. We are interested to see if the model provides a satisfactory forecast for Chinese yield curve when the financial crisis occurs.

The Chinese yield curve in the second forecasting period is relatively flat and stable at the medium and long terms while it is much more volatile at the short end. Significant fluctuations within 140 basis points can be observed at one-month interest rate over time. From February to April 2012, the whole yield curve moved up with term spread less than 35 basis points due to the relatively high CPI in the beginning of year 2012. From May to July, the treasury yields with all the maturities decrease slightly, especially at the short end and the yield curve moved down steeply. This is mostly caused by the monetary policy. The central bank cut the reserve ratio by 0.5% twice and moved down the official one-year saving and borrowing rates. In the following months to the end of 2012, the economy appears steady rise and drives the treasury yields pick up with fluctuations. In the beginning of 2013, the yield curve stays stable with flat shape. We choose this period to explore if the Fourier model can provide well prediction when the information on the instantaneous rate is not reflected on the medium- and long-term yields.

Figure 1. 3-D Plot of Yield Curve of Forecasting Period



The third forecasting period covers one year from 06/16/2014 to 06/30/2015. As shown in Figure 1, the Chinese government yield curve displays a gradually decreasing trend with stable slope from June 2014 to February 2015 and a significant decline occurred from March 2015 at the short and medium term. To keep the financing cost down and promote the development of real economy sustainable, the central bank cut down the reserve ratio three times in February, April and June 2015 by 0.5%, 1% and 0.5% respectively. At the same time, the official one-year deposit rate and loan rate were moved down four times from 3% to 2% and 6% to 4.85% within 8 months. We are interested in investigating the prediction capacity of the Fourier model when a significant change happened on the slope of the yield curve.

Table 4. Sums of 1, 5 and 21 Day Ahead Squared Forecasting Errors

	1 day ahead		5 day ahead		21 day ahead	
	Fourier	Vasicek	Fourier	Vasicek	Fourier	Vasicek
Panel A: 12/11/2008 – 12/25/2009						
1M	0.022	**0.021**	0.141	**0.122**	2.072	**1.531**
3M	**0.044**	0.073	0.163	**0.123**	2.270	**1.304**
6M	**0.056**	0.164	**0.239**	0.269	2.879	**1.782**
1Y	**0.160**	0.590	**0.280**	0.724	**2.293**	2.476
2Y	**0.838**	1.038	**0.985**	1.467	**3.121**	3.551
3Y	**0.264**	0.418	**0.481**	0.649	**2.744**	3.264
5Y	**0.189**	0.419	**0.449**	0.723	**2.199**	3.148
7Y	**0.147**	0.366	**0.384**	0.637	**1.713**	2.537
10Y	**0.105**	0.140	**0.268**	0.332	**1.078**	1.427
20Y	**0.110**	0.166	**0.156**	0.238	**0.424**	0.567
30Y	**0.126**	0.261	**0.152**	0.281	**0.251**	0.393
Sum	**2.059**	3.926	**3.697**	5.566	**21.043**	21.979
Panel B: 02/27/2012 - 03/12/2013						
1M	**0.380**	0.383	**2.012**	2.079	3.576	**3.337**
3M	**0.442**	2.603	**0.441**	2.543	**1.356**	2.738
6M	**0.185**	1.775	**0.442**	1.769	**1.911**	2.172
1Y	**0.087**	1.074	**0.227**	1.082	**1.093**	1.613
2Y	**0.128**	0.484	**0.272**	0.569	**1.121**	1.287
3Y	**0.107**	0.185	**0.259**	0.330	**1.142**	1.244
5Y	**0.072**	0.324	**0.220**	0.490	**0.701**	0.982
7Y	**0.051**	0.150	**0.100**	0.221	**0.316**	0.451
10Y	**0.162**	0.530	**0.189**	0.595	**0.304**	0.713
20Y	**0.087**	0.196	**0.111**	0.275	**0.155**	0.298
30Y	**0.035**	0.283	**0.051**	0.332	**0.087**	0.350
Sum	**1.737**	7.986	**4.324**	10.284	**11.761**	15.186

Panel C: 06/16/2014 - 06/30/2015						
1M	**0.223**	0.225	**1.604**	1.622	**5.055**	5.057
3M	**0.246**	1.054	**0.571**	1.147	**3.209**	3.589
6M	**0.119**	1.742	**0.363**	1.658	**2.947**	3.386
1Y	**0.234**	1.904	**0.612**	1.937	**3.287**	3.962
2Y	**0.109**	1.044	**0.282**	1.108	**1.748**	2.471
3Y	**0.222**	0.763	**0.322**	0.850	**1.128**	1.858
5Y	**0.132**	0.458	**0.258**	0.589	**0.979**	1.563
7Y	**0.139**	0.360	**0.256**	0.484	**0.954**	1.430
10Y	**0.233**	0.453	**0.405**	0.681	**1.327**	1.821
20Y	**0.090**	0.369	**0.237**	0.518	**0.832**	1.254
30Y	**0.106**	0.455	**0.285**	0.765	**0.965**	1.478
Sum	**1.851**	8.773	**5.195**	11.358	**22.431**	27.839

Note: Numbers in bold face denote the lower value of the sum of squared errors between models.

Table 4 gives the 1, 5 and 21-day ahead forecasting errors for the three periods as in each panel. In the period from 12/11/2008 to 12/25/2009, the aggregate sum of squared errors is reduced by 47.55%, 33.57% and 4.26% at 1, 5 and 21-day horizons by introducing the Fourier extension. The Fourier model reports close prediction error at short end of the yield curve from 1-month to 6-month, while at the medium to long end it provides much better out-of-sample forecasting than Vasicek model. In the period from 02/27/2012 to 03/12/2013, the Fourier model again delivers more accurate prediction to the observed data by largely reducing the predicting errors. Those are cut down by 78.2%, 60% and 22.6%, after incorporating the Fourier effect, at 1, 5 and 21 day ahead forecast horizons. For the forecasts of each yield through all maturities, the Fourier model wins over all the three horizons, with one exception only. At the 21-day forecasting horizon, the Fourier and Vasicek seem to provide a close performance on prediction. In the third period, the aggregate sum of squared errors is reduced by 53.6%, 36.7% and 12.4% at 1, 5 and 21-day forecasting horizons. In addition, the Fourier model outperforms the Vasicek at all maturities at all forecasting horizons.

To evaluate the predictive accuracy between the two models formally, we employ the Diebold-Mariano (DM) test[5] (Diebold and Mariano, 1995) to determine if the difference in prediction between the Fourier model and Vasicek model is statistically significant. As shown in Table 5, the DM test results indicate that there is no statistically significant difference between the two models on 21-day horizon forecasting. However, at 1 day and 5 day forecasting horizon, the Fourier model performs significantly better, especially at the medium to long end of yield curve. These findings are consistent with those by using quantitative measures in Table 4.

Table 5. The Diebold-Mariano Test

	1 day ahead	5 day ahead	21 day ahead
Panel A: 12/11/2008 – 12/25/2009			
1M	0.34	0.06	0.85
3M	-1.26	0.34	1.32
6M	-3.24***	-0.65	1.89+
1Y	-5.03***	-3.67***	-0.05
2Y	-4.64***	-3.83***	-0.78
3Y	-3.26***	-0.91	-0.83
5Y	-4.25***	-2.68**	-1.71*
7Y	-3.97***	-2.41**	-1.54
10Y	-1.54	-0.25	-0.43
20Y	-1.02	-0.35	-0.03
30Y	-3.33***	-1.21	-0.21
Panel B: 02/27/2012 - 03/12/2013			
1M	-1.38	-0.01	0.68
3M	-14.21***	-5.68***	-2.01*

5 *We use a quadratic loss function for the DM test and apply a one-side test.*

	1 day ahead	5 day ahead	21 day ahead
6M	-10.31***	-4.34***	-0.52
1Y	-9.54***	-3.74***	-1.69*
2Y	-5.29***	-2.42**	-0.24
3Y	-0.98	-0.63	-0.38
5Y	-4.63***	-2.05*	-0.90
7Y	-1.27	-1.07	-0.41
10Y	-5.26***	-3.31***	-0.78
20Y	-1.74***	-0.92	-0.08
30Y	-4.13***	-2.77**	-0.42
Panel C: 06/16/2014 - 06/30/2015			
1M	-0.76	-0.06	0.00
3M	-8.65***	-1.57	-0.67
6M	-10.32***	-4.52***	-0.89
1Y	-12.47***	-4.76***	-1.21
2Y	-9.28***	-3.65***	-1.95*
3Y	-6.43***	-3.24***	-1.71*
5Y	-4.02***	-2.56**	-1.04
7Y	-3.94***	-0.94	-0.61
10Y	-3.78***	-1.75*	-0.43
20Y	-4.61***	-2.71**	-0.21
30Y	-5.31***	-3.07**	-1.12

*Note: The Diebold-Mariano test statistics are given in this table. Negative (positive) values indicate better (worse) forecasting performance of Fourier model. The asterisks *, ** and *** represent the 5%, 1% and 0.1% statistical significance levels respectively, when the value of statistics is negative. The pluses +, ++, +++ are used for positive statistics at corresponding significance levels.*

Conclusion

In this paper, we introduce a Fourier extension of the classic Vasicek model to describe the term structure of interest rates of China. In the Fourier model, the instantaneous rate expressed by a stochastic process is assumed to revert to the long run mean which follows a Fourier series. The incorporated Fourier series is capable to describe the cyclical behaviour of interest rate in response to the government intervention to exogenous shocks. The Fourier model is found to provide more precise in-sample fitting and out-of-sample forecasting of Chinese term structure of interest rates over the original Vasicek (1977). It allows more flexibility and tractability in describing yield curve of various shapes. In addition, the superiority on prediction against the benchmark is much more significant at the medium to long end of yield curve within shorter forecasting horizons. Furthermore, it is concluded that the 2008 financial crisis had obviously influenced the Chinese term structure of interest rates.

Overall, as the establishment of interest rates liberalization in China after 30 years of reform, this paper provides a more advanced model to fit and forecast the entire yield curve of government bond markets, with excess gain of capturing the dependence of interest rates on business cycle. This model provides valuable information for central bank on practicing the countercyclical monetary policy to keep the economy humming, and help the market participants understand monetary policy information, which contained in the movements of the yield curve. The limitation of this study is that we assume the Fourier series in the model is one term only for estimation simplicity. Future work could try to extend the model with addition of the second term of Fourier series for possibly more flexibility.

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