# 8. VOLATILITY FORECASTING WITH ASYMMETRIC NORMAL MIXTURE GARCH MODEL: EVIDENCE FROM SOUTH AFRICA

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## Abstract

*This paper investigates the relative performance of the asymmetric normal mixture generalized autoregressive conditional heteroskedasticity (NM-GARCH) and the benchmarked GARCH models with the daily stock market returns of the Johannesburg Stock Exchange, South Africa. The predictive performance of the NM-GARCH model is compared against a set of the GARCH models with the normal, the Student-t, and the skewed Student-t distributions. The empirical results show that the NM-GARCH outperforms all other competing models according to Christoffersen's (1998) tail-loss and White's (2000) reality check tests. This evidence shows that mixture of errors improves the predictive performance of volatility models.*

**Keywords:** volatility forecasting, value-at-risk, asymmetric normal mixture GARCH, reality check.

**JEL Classification**: C32, C53, G17

## 1. Introduction

Value-at-risk (VaR) became the standard benchmark for measuring risks in volatility forecasting. The VaR become popular in the 1990s, following well-known disasters such as Orange Country, Barings, Metallgesellschaft, Dawia, and many others. The global financial crises that began in 2007s have renewed concerns about risk forecasting. The VaR is usually estimated with Bollerslev's (1986) generalized autoregressive conditional heteroskedasticity (GARCH) model. During the last 25 years, different GARCH-based models that consider asymmetry, long memory, structural breaks, and regime switching behaviors in the data have been developed.

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Among others, the exponential GARCH (Nelson, 1991), the GJR-GARCH (Glosten et al., 1993), and the fractionally integrated GARCH (Baillie et al., 1996; Chung, 1999) are the most well-known extensions of the GARCH models that used for volatility forecasting. Haas et al. (2004) and Alexander and Lazar (2003) have proposed the normal mixture GARCH which is a combined model and the errors have a normal mixture conditional distribution. Alexander and Lazar (2004, 2006) investigate asymmetric NM-GARCH model where the error term follows a normal mixture distribution. This model captures the persistent asymmetry according to the different means in conditional normal mixture distributions and the dynamic asymmetry according to the skewed GARCH process. The normal mixture GARCH models are flexible in single variance process and have time-varying conditional higher moments compare to normal GARCH model. Alexander and Lazar (2006) apply the asymmetric normal mixture GARCH model to the volatility of the major exchange rates. They find that the asymmetric normal mixture GARCH model should be used instead of the Student-t GARCH model in order to capture the leptokurtosis in the financial data. Alexander and Lazar (2009) introduce the two-state asymmetric normal mixture GARCH model and apply this model to European equity indices. They find that the two-state asymmetric normal mixture GARCH model is the best one compare to fifteen conventional GARCH models. Cifter and Ozun (2007) and Drakos et al. (2010) test the predictive performance of the asymmetric normal mixture GARCH model. They find that none of the models including the normal mixture GARCH is appropriate for both long and short trading positions. Moreover, they state that the asymmetric normal mixture GARCH model increases the predictive performance for Turkish and Greek equity markets.

Forecasting the prices and the volatility of emerging markets is a critical task for the local and international investors. The Johannesburg Stock Exchange (JSE) is the largest and the most developed emerging market in Africa (Hearn and Piesse, 2009). At the end of December, 2010, the market capitalization of the JSE is $925 billion and this largest African stock market holds a treasured position as one of the top 20 exchanges in the World[2]. Besides, the JSE constitute a reform process in late 1995 to allow greater foreign investment in South Africa (McMillan and Thupayagale, 2008). The volatility of the South African stock market has been forecasted with different GARCH models with asymmetric, long memory, and structural break effects. The GARCH-based models actually contradict weak-form efficiency theory. According to weak-form efficiency, future prices cannot be predicted by analyzing the previous prices. Therefore, if a market is weak-form efficient, the GARCH models will be inappropriate for volatility forecasting. An early study of Roux and Gilbertson (1978) show that the JSE is not weak-form efficient over the periods 1971-1976. Appiah-Kusi and Menyah (2003) use the EGARCH-M model to test the weak-form efficiency of African stock markets. They find that the South African stock market is not weak-form efficient similar to Roux and Gilbertson's (1978) findings. Jefferis and Smith (2005) test the efficiency of African stock markets using a GARCH approach with time-varying parameters. In contrast to previous studies, they find that the JSE market is weak-form efficient, which shows that the JSE market has neither a long nor a short

[2] Source: http://www.jse.co.za/How-To-List-A-Company/Main-Board.aspx

memory. McMillan and Thupayagale (2008) examine long memory in the JSE all-share returns using the ARFIMA-FIGARCH model to determine the efficiency of the market. They find that the behavior of volatility in South Africa could be efficiently forecasted if long memory is considered, which might result in an improvement in volatility forecasts. Jefferis and Thupayagale (2008) also examine long memory effect in the South African stock market using the ARFIMA-FIGARCH model and find that using past information improves the predictability of future volatility. Some studies estimate the volatility in the JSE market with structural breaks and combined GARCH models. Babikir et al. (2010) investigate the structural breaks in forecasting stock return for the JSE. They find that structural breaks are empirically relevant to stock return volatility in the South African stock market. Seymour and Polakow (2003) estimate the value-at-risk models with a combined GARCH model approach and extreme value theory model for the portfolio of the South African stocks. They find that the combined GARCH-based model provide significantly better results than the normal GARCH models.

Backtesting of volatility models is as important as the choice of volatility models. The best volatility forecasting model can be selected with different type of backtesting procedures. The predictive performances of GARCH-based models are compared with various backtesting procedures. In an earlier study, Chong et al. (1999) use goodness-of-fit statistics such as mean squared error. Wong et al. (2003) compare the predictive performance of ARCH- and GARCH-based models with Basel Committee criteria as the number of vialotions. They find that using ARCH- and GARCH-based models in VaR estimation is not a reliable way to manage a bank's market risk. Christoffersen (1998) developed tail loss test and Cifter and Ozun (2007), Chen et al. (2009), Siu and Okunev (2009), Dunis et al. (2010), Drakos et al. (2010), and Cifter (2011) use this test to compare the performance of GARCH-based models. Since tail loss test does not consider cumulative failure probability, Gonzales-Rivera et al. (2004), Hansen and Lunde (2005), Souza et al. (2005), Marcucci (2005), Bao et al. (2006), and Laurent and Violante (2011) use White's reality checks. Among others, Cifter and Ozun (2007), Alexander and Lazar (2003, 2004, 2006, 2009), and Drakos et al. (2010) use the NM-GARCH-based models and compares the predictive performance of volatility models with the RMSE, number of vialotions, Kupiec's (1995) and Christoffersen's (1998) tail loss tests. This paper differs from previous studies in that it uses White's (2000) reality check to compare the predictive performance of the NM-GARCH model against benchmarked GARCH-based models. The NM-GARCH model has an advantage compare to other GARCH-based models that it can capture time variation in both conditional skewness and kurtosis by a mixture of normal distribution. The empirical results show that the NM-GARCH outperforms all other competing models according to the tail-loss and the reality check tests. Besides, the asymmetric normal mixture GARCH with skewed student-t distribution is found to be the most accurate model and this also shows that selecting the distribution is one of the major factors in the performance of volatility models. This evidence shows that mixture of distributions improves the predictive performance of volatility models.

The remainder of the paper is organized as follows. Section 2 provides the asymmetric normal mixture GARCH and the backtesting methodologies. Section 3 describes the data on daily index returns. Section 4 presents empirical results for the forecasting performance of the models and the final section concludes the study.

## 2. Methodology

The VaR measures the worst expected loss over a given horizon at a given confidence level (Jorion, 2001, p.22). The VaR is estimated with conditional variance of stock returns. Let $p_t$ denotes stock prices and $r_t$ denotes its corresponding rate of return:

$$r_t = [\log(p_t) - \log(p_{t-1})] \tag{1}$$

in which: *t* denotes daily closing observations. The return series can be converted with the following conditional mean equation:

$$r_t = \mu + \varepsilon_t, \qquad \varepsilon_t = z_t \sigma_t \tag{2}$$

in which: $\mu$ is the conditional mean, $z_t$ is independent and identically distributed with $N(0,1)$, and $\sigma_t^2$ is the conditional variance. $\sigma_t^2$ can be estimated with GARCH-based models. In this paper, the VaR is estimated with the normal mixture GARCH and the predictive performance of this model is bencmarked against the Riskmetrics-EWMA, GARCH (1,1), asymmetric GARCH, and the fractionally integrated GARCH models. The Riskmetrics-EWMA is the simplest volatility forecasting model developed by J.P.Morgan. This model can be defined as a special case of the GARCH model, where the ARCH parameter $\alpha = 1 - \lambda$, the GARCH parameter $\beta = \lambda$, and the constant term $\alpha_0 = 0$. The Riskmetrics-EWMA variance model can be written as (J. P. Morgan, 1994):

$$r_t = \mu_t + \varepsilon_t,$$
$$\varepsilon_t = \sigma_t z_t,$$
$$\sigma_t^2 = \lambda \sigma_{t-1}^2 + (1-\lambda)\varepsilon_{t-1}^2 \tag{3}$$

in which, the conditional variance $Var(r_t | \psi_t) = \sigma_t^2$ is non-constant, and $\lambda$ is the decay factor that determines relative weights. $\lambda$ is usually set to 0.94 for daily data (J.P.Morgan, 1996). The higher the decay factor, the longer the previous returns. In this paper, $\lambda$ is set to 0.94 since daily stock returns is used.

The GARCH model is proposed by Bollerslev (1986) by extending Engle's (1982) ARCH model for time varying volatility in a time series. The GARCH model can be shown as:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{4}$$

Where: $\alpha_0$ is the constant term, $\varepsilon_t / \psi_{t-1} \sim N(0, \sigma_t)$ as $N(.)$ is a probability density function with mean (0) and conditional variance ($\sigma_t^2$), and $\sigma_t$ is conditional volatility of $\varepsilon_t$ with the conditions of $\alpha, \beta > 1$ and $\alpha_0 > 1$.

The asymmetric GARCH model is developed by Engle (1990) to capture asymmetric volatility response in the GARCH process. Glosten et al. (1993) proposed the GRJ-GARCH(1,1) model, in which asymmetric response weight is differentiated for negative and positive shocks. The GJR-GARCH (1,1) model can be estimated as (Glosten et al., 1993):

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \gamma 1 \varepsilon_{t-1}^2 I_{t-1} + \beta_1 \sigma_{t-1}^2 \tag{5}$$

Where: $I_{t-1} = 1$, if $\varepsilon_{t-1} < 1$ and $I_{t-1} = 0$ , otherwise. Therefore, the impact of $\varepsilon_{t-1}^2$ on $\sigma_t^2$ depends on the sign of $\varepsilon_t$ . The positive news affects $\alpha_1$, while the negative news affects $\alpha_1$ and $\gamma 1$. The normal GARCH models consider only short-term volatility therefore these models are inadequate for long-range volatility dependence. Baillie et al. (1996) propose the fractionally integrated GARCH model that considers the long-memory properties for volatility estimation. The FIGARCH model is defined as:

$$\phi(L)(1-L)^d \varepsilon_t^2 = \alpha_0 + [1-\beta(L)]\varepsilon_t^2 - \sigma_t^2 \tag{6}$$

$d$ determines the fractional order parameter. If $d = 0$ , this model become a normal GARCH process, and if $d = 1$, this model become an integrated GARCH process. For $0 < d < 1$ , the conditional variance exhibits long memory properties. The conditional variance of the FIGARCH model can be estimated as (Baillie et al., 1996):

$$\sigma_t^2 = \alpha_0 + [1-\beta(L)]^{-1} + \{1-[1-\beta(L)]^{-1}\phi(L)(1-L)^d\}\varepsilon_t^2$$

(7)

Chung (1999) proposed a new FIGARCH model since Baillie et al.'s (1996) model may have specification problem on $\alpha_0$. Chung (1999) defined the conditional variance of the FIGARCH model as:

$$\sigma_t^2 = \sigma_{t-1}^2 + \{[1-\beta(L)]-1\phi(L)(1-L)d\}(\varepsilon_t^2 - \sigma_{t-1}^2) \tag{8}$$

In this paper, the FIGARCH model is estimated with Chung's (1999) suggestion. Following Engel (1990), Nelson (1991), and Glosten et al. (1993), many alternative GARCH models with leverage effects are developed. Nevertheless, none of the conventional GARCH models can capture time variation in conditional skewness or kurtosis. Hass et al. (2004) and Alexander and Lazar (2003) have introduced the normal mixture GARCH model, in which errors have a normal mixture conditional distribution. Alexander and Lazar (2004, 2006, 2009) extended the normal mixture GARCH to the asymmetric normal mixture GARCH to capture the lasting asymmetry according to the different means in conditional normal mixture distributions and the dynamic asymmetry according to the skewed GARCH process. The asymmetric normal mixture GARCH model has mean ($\mu$) and *K* conditional variance components. For simplicity, the conditional mean equation is shown as $y_t = \varepsilon_t$ assuming that there are no explanatory variables. The error term $\varepsilon_t$ that captures the market shock is assumed to have a conditional normal mixture. The *K* normal density functions with different means and variances is given as:

$$\sum_{i=1}^{K} p_i = 1, \quad \sum_{i=1}^{K} p_i \mu_i = 1$$

$$\varepsilon_t / \psi_{t-1} \sim NM(p_1,....., p_K, \mu_1,....., \mu_K, \sigma_{1t}^2,......, \sigma_{Kt}^2), \tag{9}$$

and the conditional density of the error term is derived as:

$$\eta(\varepsilon_t) = \sum_{i=1}^{K} p_i \varphi_i \tag{10}$$

in which: $\varphi$ represents normal density functions with different means $\mu_i$ and different time-varying variances $\sigma_{it}^2$ for *i = 1,…, K*.. The NM-GARCH (1,1) is estimated as (Alexander and Lazar, 2009):

$$\sigma_{it}^2 = \alpha_0 + \alpha_i \varepsilon_{t-1}^2 + \beta_i \sigma_{it-1}^2 \qquad \textit{for i=1,....,K;} \tag{11}$$

The asymmetric normal mixture GARCH (NM-GJR GARCH) based on Glosten et al, (1993) is given as (Alexander and Lazar, 2009):

$$\sigma_{it}^2 = \alpha_0 + \alpha_i \varepsilon_{t-1}^2 + -\lambda_i d_{t-1}^- \varepsilon_{t-1}^2 + \beta_i \sigma_{it-1}^2 \quad \textit{for i=1,....,K;} \tag{12}$$

Where: $d_t^- = 1$ if $\varepsilon_t < 0$ , and 0 otherwise. Besides, the parameters are estimated as $\alpha_i > 0$ and $1 > \beta_i \geq 0$. Alexander and Lazar (2004) showed that the condition of $\alpha_i + \beta_i < 0$ is not required in every case. For both models, the overall conditional variance is defined as:

$$\sigma_t^2 = \sum_{i=1}^{K} p_i \sigma_{it}^2 + \sum_{i=1}^{K} p_i \mu_i^2 \tag{13}$$

This paper applies the NM-GJR GARCH model for normal mixture volatility estimation to capture asymmetric effects. Following Alexander and Lazar (2006), the NM-GARCH model where the errors have normal mixture conditional distributions with GARCH variance components is estimated. The NM-GARCH model can be estimated with Gaussian (Bollerslev, 1986), student-t (Bollerslev, 1987), and skewed Student-t (Fernandez and Steel, 1998) distributions. In this paper, the asymmetric normal mixture GARCH and other GARCH models are estimated with these three distributions.

In this study, out-of-sample forecasting performance of the conditional volatility models are evaluated with the root mean squared error (RMSE), Christoffersen's (1998) conditional coverage test, and White's (2000) reality check. The main drawback of number of violations and RMSE approachs is that they consider neither failure rate nor cumulative failure probability. On the other hand, Christoffersen's (1998) test is used as failure probability and White's (200) test is used for multiple forecast comparision. Christoffersen's (1998) conditional coverage and White's (2000) reality check have superior forecasting performance for the one-day-ahead VaR estimation.

Christoffersen (1998) proposed the likelihood ratio (LR) test of conditional coverage that shows whether the VaR models have a correct coverage at each point in time.

The conditional coverage test consists of two tests, for unconditional coverage and independence. The LR test of conditional coverage can be estimated as:

$$LR_{cc} = -2In\left[(1-p)^{T_0} p^{T_1} / \left\{(1-T_1/T)^{T_0} (T_1/T)^{T_1}\right\}\right] \tag{14}$$
$$-2In\left[(1-T_1/T)^{T_0} (T_1/T)^{T_1} / \left\{1-\hat{\pi}_{01})^{T_{00}} \hat{\pi}_{01}{}^{T_{01}}\right\}\right] \sim \chi_2^2$$

Where: $T_0$ and $T_1$ are the number of 0s and 1s in the sample, $p$ is the VaR's theoretical coverage rate $\alpha$, $T$ is the total number of observations, and $\hat{\pi}_{01} = T_{01}/(T_{00}+T_{01})$. The first part of the test is the unconditional coverage test ($LR_{uc}$), and the second part is the independence ($LR_{ind}$) test. The drawback of Christoffersen's (1998) conditional coverage test is that the predictive performance of other models is not compared with the tested model. On the other hand, White's (2000) reality check (RC) compares the predictive performance of each of the model against the alternative models. The RC test is the first and the most important superior predictive ability (SPA) test[3]. White's reality check compares $l+1$ forecasting models and the null hypothesis is that none of the models $k=1,....,l$ outperforms the benchmank model (model 0, $k_0$). In this paper, $l=12$ plus a benchmark model is considered. Assuming that the loss from the conditional volatility model is defined as $L_t = L(\hat{\sigma}_t^2 - \sigma_t^2)$, where $\hat{\sigma}_t^2$ is the realized volatility and $\sigma_t^2$ is the predicted volatility, the performance of k models relative to the bechmarked model can be defined as

$$f_{k,t} = L_{t,0} - L_{t,k}, \tag{15}$$

Where: $L_{t,0}$ is the loss from the benchmarked model, and and $L_{t,k}$ is the loss from the alternative models. In this paper, the loss function is estimated with the mean squared error, $MSE = \frac{1}{n}\sum_{t=T+1}^{T+n}(\hat{\sigma}_t^2 - \sigma_t^2)$ , with 20 days rolling window ($n = 20$)[4]. The null hypothesis is that none of the models is better than the benchmarked model, and formally can be shown as

$$H_0 = \max_{k=1,....,l} E(f_k^*) \leq 0. \tag{16}$$

Where: $E(f_k^*)$ is the expected relative performance of model k relative to the benchmark model. The alternative hypothesis ($H_1$) is that the best model is superior to the benckmarked model. We can estimate $E(f_k^*)$ with the sample average $\bar{f}_{k,n} = n^{-1}\sum_{t=1}^{n} f_{k,t}$ and obtain the bootsrap RC test statistics as

$$T_n = \max_{k=1,....,l} n^{1/2} f_{k,n} \tag{17}$$

[3] Marcucci (2005) reported that testing for the SPA is certainly more relevant than testing for equal predictive ability (EPA), such as the MSE tests.

[4] 20 days represent one month rolling window.

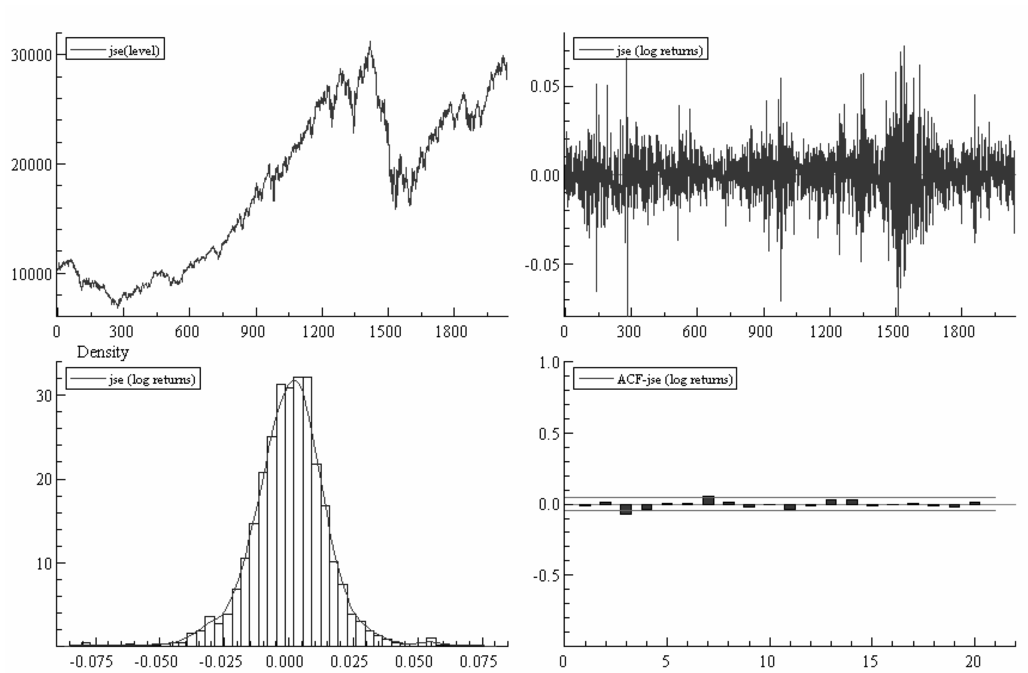
If we reject the null hypothesis, this shows that at least one model is significantly better than the benchmark model. Following White (2000), the bootsrap RC is implemented[5] with the stationary bootsrap of Politis and Romano (1994). This paper estimates the VaR value at 1% confidence level in line with Basel II requirements. The VaR forecasting performances are compared to the number of violations, the root mean squared error (RMSE), the LR and the RC tests for the left tail.

## 3. Data

The data consists of closing prices of the Johannesburg Stock Exchange (JSE) all-share index. The series is from Bloomberg, sampled at a daily frequency. The dataset covers 2283 daily observations from February 7, 2002 to March 11, 2011[6]. The Daily JSE index level observations, log returns, histogram and autocorrelation graphs are shown in Fig. 2. From these figures, it can be inferred that there are extreme observations in the JSE index and that according to the autocorrelation graph, the returns are not correlated.

**Figure 1**

**Daily JSE index level observations, log returns, histogram and autocorrelation graphs**




[5] White (2000) suggests two procedures, namely the "monte carlo reality check", and the "bootsrap reality check". This paper uses the bootsrap reality check.

[6] The first 275 observations are used to estimate the starting parameters in the GARCH models.

In order to test volatility effects, a time series should not contain a unit root. The augmented Dickey–Fuller (Dickey and Fuller, 1981) and Philips-Perron (Philips and Perron, 1988) tests are general unit root tests. Both of these tests have size and power weaknesses, we also checked the stationary properties of the stock returns with the Dickey-Fuller Test with GLS detrending-ADF-GLS (Elliott et. al. 1996) and the Ng-Perron (Ng and Perron, 2001) tests. Table 1 reports the unit root test results, and all the tests indicate that stock returns is stationary; therefore, univariate volatility models are set based on log-differenced series.

**Table 1**

**Unit root test statistics of the time series**

| | ADF test | P-P test | ADF-GLS test | Ng-Perron tests | | | |
|---|---|---|---|---|---|---|---|
| | | | | $MZ_\alpha$ | $MZ_t$ | MSB | MPT |
| Jse | -0.719 | -0.562 | 0.391 | 0.537 | 0.391 | 0.727 | 36.919 |
| lnJse | -45.623* | -45.681* | -4.941* | -17.484* | -2.906* | 0.166* | 1.591* |

*Notes: Tests contain a constant but not a time trend. The number of lags has been selected using the Schwarz information criterion with a maximum of twelve lags. * Indicate the rejection of the unit root null at least 5% significance level.*

The descriptive statistics for JSE returns are provided in Table 2. It is observed that the JSE returns is far from normal distribution according to the Jarque-Bera (Jargue and Bera, 1980) test, and skewness and excess kurtosis values. Besides, lagrange multiplier (LM) statistics of Engle (1982) indicates that ARCH effects exist for JSE index and this shows that the JSE market should be forecasted with the conditional volatility models.

**Table 2**

**Descriptive Statistics**

| | JSE | Diagnostics | Estimates |
|---|---|---|---|
| Mean | 0.000281 | ARCH tests* | |
| Std.Deviation | 0.014892 | LM[2] | 92.662<br>(0.000) |
| Skewness | -0.18685 | LM[5] | 75.825<br>(0.000) |
| Excess Kurtosis | 3.3476 | LM[10] | 45.48<br>(0.000) |
| Minimum | -0.079683 | | 3.3476 |
| Maximum | 0.072491 | | |
| Jarque-Bera test | 963.94<br>(0.000) | | |

*Notes: Numbers in brackets are p-values. * Lag-lengths are shown in squared brackets.*

## 4. Empirical Evidence

In this section, the relative performance of the NM-GARCH and the benchmarked GARCH models are illustrated. The Riskmetrics-EWMA, GARCH (1,1), GRJ-GARCH(1,1), and FIGARCH (1,1) models are selected as benchmarked models. Table 3 shows the parameters estimated from GARCH (1,1) models. According to Akaike (AIC) selection criteria, the NM-GARCH model outperforms the other GARCH models but this evidence should be tested with appropriate backtesting procedure. Both the alpha ($\alpha$) and beta ($\beta$) parameters are statistically significant in the GARCH (1,1) models, and this evidence indicates that the JSE returns have ARCH and GARCH effects. The asymmetry parameters ($\gamma 1, \gamma$) and the long memory parameter (*d*) are statistically significant at the 5% critical level and this result provides evidence of the asymmetry and long memory effects in the JSE returns. The Student-t parameter ($v$) is statistically significant for all the GARCH models; therefore it can be inferred that the return series are fat-tailed. Besides, the skewed Student-t parameter ($\xi$) is negative and statistically significant; therefore this result indicates that the return series are skewed to the left and it is expected that the GARCH models with the skewed Student-t distribution would have better forecasting performance than the Gaussian and the Student-t distributions.

**Table 3**

**Estimation results from GARCH (1,1) models**

| Model | $\alpha_0^{**}$ | $\alpha$ | $\beta$ | $v$ | $\xi$ | $v$ - Skew | *d* Figarch | $\gamma 1$ - GRJ | $\gamma$ -NM | AIC |
|---|---|---|---|---|---|---|---|---|---|---|
| Riskmetrics-EWMA | 0.76* (2.45) | 0.06 | 0.94 | - | - | - | - | - | - | -5.80 |
| GARCH (1,1)-n | 0.79* (2.69) | 0.10* (6.80) | 0.90* (76.09) | - | - | - | - | - | - | -5.81 |
| GARCH (1,1)-t | 0.88* (2.99) | 0.09* (6.37) | 0.91* (73.86) | 13.0* (3.29) | - | - | - | - | - | -5.82 |
| GARCH (1,1)-skew | 0.79* (2.72) | 0.09* (6.61) | 0.91* (77.15) | - | -0.1* (3.30) | 13.7* (3.75) | - | - | - | -5.82 |
| GRJ GARCH (1,1)-n | 0.62* (2.07) | 0.04* (2.48) | 0.91* (66.99) | - | - | - | - | 0.082* (4.01) | - | -5.82 |
| GRJ GARCH (1,1)-t | 0.79* (2.61) | 0.03* (2.09) | 0.92* (69.19) | 14.2* (3.57) | - | - | - | 0.085* (4.28) | - | -5.83 |
| GRJ GARCH (1,1)-skew | 0.73* (2.44) | 0.03* (2.16) | 0.92* (74.18) | - | -0.1* (3.58) | 15.6* (3.29) | - | 0.08* (4.51) | - | -5.83 |
| FIGARCH (1,1)-n | 0.79* (2.56) | 0.13* (2.99) | 0.78* (16.90) | - | - | - | 0.724* (10.28) | - | - | -5.81 |
| FIGARCH (1,1)-t | 0.86* (2.87) | 0.14* (3.12) | 0.77* (15.56) | 14.9 * (4.23) | - | - | 0.703* (9.58) | - | - | -5.82 |
| FIGARCH (1,1)-skew | 0.84* (2.87) | 0.12* (2.78) | 0.78* (15.56) | - | -0.1* (3.47) | 15.6* (3.95) | 0.71* (9.58) | - | - | -5.82 |

| Model | $\alpha_0^{**}$ | $\alpha$ | $\beta$ | $v$ | $\xi$ | $v$ - Skew | d Figarch | $\gamma 1$ - GRJ | $\gamma$ -NM | AIC |
|---|---|---|---|---|---|---|---|---|---|---|
| NM-GRJ GARCH1,1)-n | 0.58* (1.82) | 0.05* (2.49) | 0.91* (58.65) | - | - | - | - | - | 0.08* (3.70) | -5.82 |
| NM-GRJ GARCH ( 1,1)-t | 0.78* (2.40) | 0.04* (2.07) | 0.91* (59.63) | 12.1* (3.92) | - | - | - | - | 0.08* (3.93) | -5.83 |
| NM-GRJ GARCH (1,1)-skew | 0.70* (2.17) | 0.04* (2.21) | 0.92* (64.76) | - | -0.1* (3.49) | 13.3 * (3.58) | - | - | 0.07* (4.08) | -5.83 |

*Notes: * 5% confidence level. t-statistics are shown in brackets. * * The estimated parameter constant in mean is multiplied by 1000.*

Table 4 shows the backtesting of the VaR models for the number-of-violations, RMSE, Christoffersen (1998) tail-loss, and White's (2000) bootsrap reality check tests. According to the number of violations test, the NM-GARCH model the best one. According to RMSE criteria, however, the the Riskmetrics-EWMA is the best model, and the NM-GARCH is the worst. Since the tail-loss and reality check tests are more appropriate, the predictive performance of the GARCH models should be tested with those tests. Christoffersen (1998) tail-loss test shows that that the NM-GARCH outperforms all other competing models, since the lowest p-values belongs to these model. Same as tail-loss test, White's (2000) bootsrap reality check shows that the NM-GARCH with the skewed Student-t distribution is the best model compare to the other volatility models. Only the FIGARCH (1,1) with the Skewed student-t distribution has better forecasting performance than the NM-GARCH (1,1) with the normal and the Student-t distribution, but not the NM-GARCH (1,1) with the skewed Student-t distribution. This evidence also shows that the NM-GARCH model should be estimated with skewed Student-t distribution. The comparison of the GARCH models' out-of-sample forecasting graphs is shown in Figure 2. This graph indicates that NM-GARCH models capture the fat-tailed behavior better than the benchmarked GARCH models. This evidence shows that mixture of errors improves the predictive performance of volatility models. Figure 3 shows the tail-loss ($LR_{CC}$) versus the reality check (RC) tests. This figure indicates that the $LR_{CC}$ and the RC tests have approximately same value for Riskmetrics-EWMA and GARCH models, but the statistics values vary for other models.

**Table 4**

**Backtesting of VaR Models**

| Model | No. of violations* | RMSE | $LR_{CC}$** | RC*** |
|---|---|---|---|---|
| Riskmetrics-EWMA | 31 | 0.0371 | 38.282 | 0.0039 |
| GARCH (1,1)-n | 30 | 0.0376 | 37.457 | 0.0038 |
| GARCH (1,1)-t | 30 | 0.0370 | 37.251 | 0.0038 |
| GARCH (1,1)-skew | 28 | 0.0392 | 32.427 | 0.0039 |
| GRJ GARCH (1,1)-n | 30 | 0.0375 | 24.519 | 0.0039 |

| Model | No. of violations* | RMSE | $LR_{CC}$** | RC*** |
|---|---|---|---|---|
| GRJ GARCH (1,1)-t | 27 | 0.0385 | 23.705 | 0.0036 |
| GRJ GARCH (1,1)-skew | 25 | 0.0399 | 12.236 | 0.0020 |
| FIGARCH (1,1)-n | 28 | 0.0377 | 25.340 | 0.0030 |
| FIGARCH (1,1)-t | 25 | 0.0387 | 23.705 | 0.0023 |
| FIGARCH (1,1)-skew | 22 | 0.0397 | 13.956 | 0.0006 |
| NM-GRJ GARCH 1,1)-n | 21 | 0.0391 | 6.654 | 0.0018 |
| NM-GRJ GARCH ( 1,1)-t | 19 | 0.0403 | 3.572 | 0.00017 |
| NM-GRJ GARCH (1,1)-skew | 18 | 0.0409 | 2.559 | 0.00000 |

*Notes: * Number of violations for left tail. ** The $LR_{CC}$ represents Christoffersen's (1998) conditional coverage test based on $\chi^2_2$ test with %1 confidence level. *** The RC represents White's (2000) bootsrap reality check with 1000 bootsrap replications where the loss function is estimated with the mean squared error (MSE).*

**Figure 2**

**Comparison of the VaR Models at $\alpha = 0.01$**

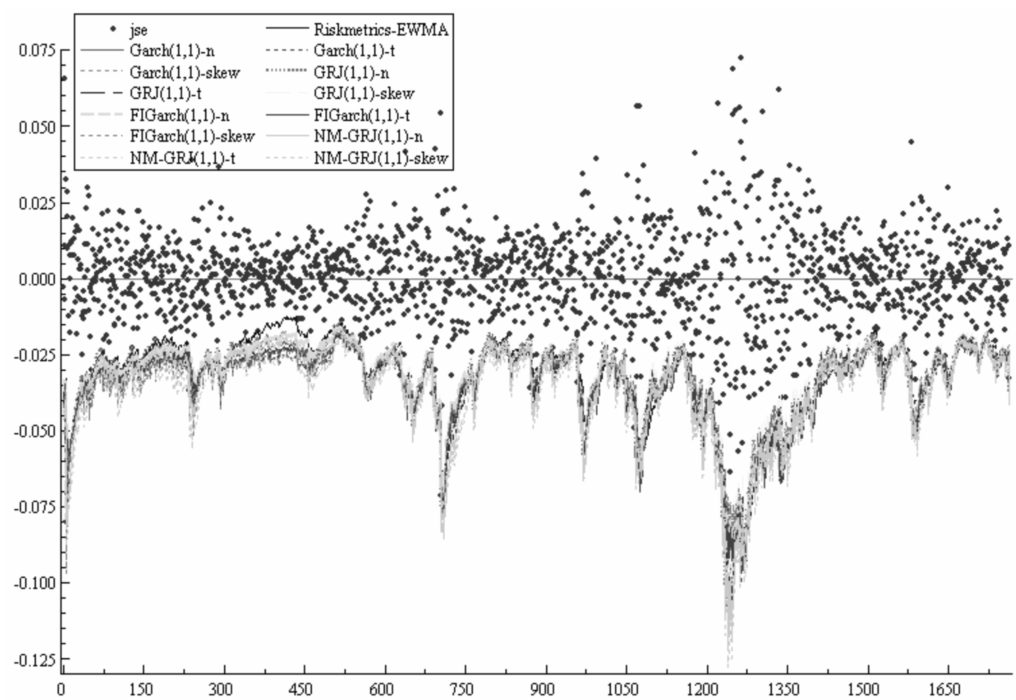
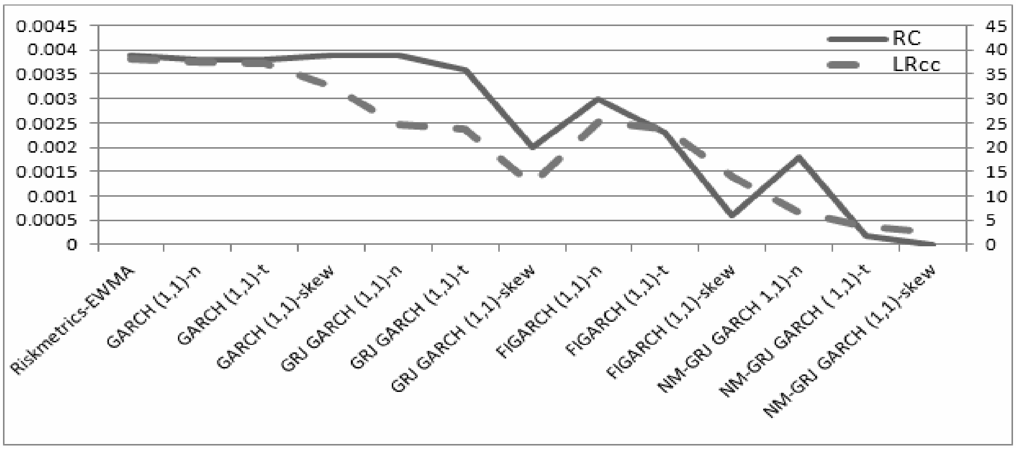


**Figure 3**

**The Tail-loss ($LR_{CC}$ ) versus The Reality Check (RC) tests**




## 5. Conclusion

This paper investigates the relative performance of asymmetric normal mixture generalized autoregressive conditional heteroskedasticity (NM-GARCH) and the benchmarked GARCH models with daily stock market returns from the JSE, South Africa. The NM-GARCH model is benchmarked against the Riskmetrics-EWMA, the GARCH, the asymmetric GARCH, and the fractionally integrated GARCH models with normal, Student-t, and skewed Student-t distributions. The main advantage of the NM-GARCH model is that it can capture time variation in both conditional skewness and kurtosis by a mixture of normal distribution. The predictive performance of volatility models is compared with RMSE, Christoffersen's (1998) tail-loss, and White's (2000) bootsrap reality check tests. It is found that the NM-GARCH (1,1) models significantly reduces the number of violations, which is an important factor in determining the VaR by the financial as well as non-financial institutions. According to the tail-loss and the reality check tests, the NM-GARCH model outperforms the benchmarked models. Besides, the NM-GARCH with skewed student-t distribution is found to be the most accurate model, where FIGARCH (1,1) with the skewed Student-t distribution is found to be the second best model according to the reality check test. The empirical evidence shows that the NM-GARCH(1,1) with the skewed Student-t distribution improves the predictive performance of volatility models with the mixture of distributions.

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