6. A VIEW ON THE RISK-NEUTRAL DENSITY FORECASTING OF THE DAX30 RETURNS

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Abstract

Option-implied risk-neutral densities incorporate market expectations with respect to the future course of option underlyings. Under the risk neutrality assumption various methods have been developed. In this paper, we look into two of them: parametric mixture of lognormals method and non-parametric Rookley method. We use option data on the DAX30 index. The Berkowitz test is employed to check the goodness of fit of the estimated densities, while we use the likelihood criterion to compare their performance. Our results show that risk-neutral densities can be good predictors for horizons of 4, 5 and 6 weeks, while for horizons of 2 and 3 weeks the null hypothesis that risk neutral densities are accurate predictors of the distribution of the DAX30 returns is rejected.

Keywords: risk neutral density, mixture of lognormals, kernel smoothing in implied volatility space, option prices

JEL Classification: C13, C14, D53, G12

I. Introduction

Forecasts of future asset prices lie in the interest of agents acting on the financial markets, financial institutions and, nevertheless, central banks. A forecast of asset prices can be obtained from option prices. Option data provide valuable information for obtaining point forecasts of future asset prices and, what is more important, for estimating the distribution over possible values of the asset price. This is possible through the very nature of options: prices of options today reflect what the market expects to happen with respect to the price of the option underlying at the maturity of the option. The fact that for a particular underlying, option data is available for different strikes enables us to estimate a density function of the underlying. As option pricing is done under the risk neutral measure, the probability density implicitly contained in option prices is the risk neutral density (RND). In the literature it might also be found

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under the name of state price density. One must note that the RND incorporates not only investors' expectations, but also investors' risk premia. Therefore, they should not be confused with real-world or physical densities. In this paper we focus on the estimation of RND.

Several methods to estimate the RND were developed. A general classification might separate them into two classes: parametric and non-parametric. The parametric models typically rely on particular assumptions, but have the advantage of finally rendering an analytical form of the risk-neutral density. Probably the most popular models in this category are the Black and Scholes (1973) and Heston (1993) models. These two models make assumptions over the stochastic process characterizing the option underlying. Another category of parametric models assumes that the RND can be approximated by a mixture of densities. This is less restrictive, in the sense that a particular RND can be consistent with several stochastic processes, whereas a stochastic process is consistent with only one RND. Melick and Thomas (1997) specify the density of the option underlying as a mixture of three lognormal distributions. They constructed a model that can be applied to American options, meaning that it also deals with the early exercise feature of American options. Bahra (1997) uses a mixture of two lognormals to recover the risk neutral density from European type options.

In contrast, non-parametric methods are more flexible, data-driven methods on one hand, but on the other hand the resulting densities do not have an analytical form. There are several different non-parametric methods that were approached: the implied binomial trees introduced by Rubinstein (1994), the smoothing kernel regression techniques employed by Rosenberg and Engle (2002), smoothing in implied volatility space presented in Bliss and Panigirtzoglou (2002) or Rookley (1997) or the positive convolution approximation technique developed by Bondarenko (2003). Grith *et al.* (2011) compare different non-parametric estimation techniques and express a preference for Rookley (1997).

This list of possible estimation techniques for the risk-neutral density is not at all exhaustive. Even so, the area of choice is broad, so a natural question that arises is which method should be preferred? Parametric or non-parametric? Bliss and Panigirtzoglou (2002) argue that smoothed implied volatility smile RND estimation is preferred to the mixture of two lognormals method in terms of stability of the probability density function (PDF) estimates. Bondarenko (2003) compare their positive convolution approximation (PCA) method with a number of other methods including lognormal mix with one, two and three components. They find that mixes with more than three components perform unsatisfactorily in comparison with PCA. Alonso *et al.* (2005) find no significant difference between employing a mixtureof two lognormals and a spline smoothing in implied volatility space method. Moreover Liu *et al*. (2007) find that the spline estimated RND underperforms in comparison to the two-component mixtureof lognormals.

In this context, one of the objectives of this paper is to check how the parametric two mixturecomponent method performs in comparison to the non-parametric smoothing technique in implied volatility space of Rookley (1997). To achieve this, we will use ODAX options traded on EUREX from 2002 to 2011. At the same time we investigate whether the RND estimates fit the actual realizations of the DAX30 (which is the underlying of the ODAX options) in the sample period. This is of particular use for stakeholders employing RNDs for further inference relevant to monetary policy or assessing market conditions.

Having stated our objectives, we proceed in the second section with introducing the two estimation methods as well as the testing methodology employed in this study. In the third section a description of the data is provided. The forth section is dedicated to empirical results, while in the fifth section we conclude.

2. Estimation and Testing Methods

2.1 Mixture of Two Lognormals

The first estimation method to be introduced lies in the category of parametric methods. We assume a functional form for the RND and then we estimate the parameters characterizing this function by minimizing the squared difference between the actual call and put option prices and the price functions generated by assuming a particular form of the underlying density.

Empirical evidence shows that option underlyings (e.g. market indexes, equities) display implied negative skewness. This is documented in a constantly increasing number of papers in the finance literature, see, e.g., Christofferson *et al*. (2006). Considering this evidence, the RNDs should be able to capture this feature. The literature suggests different parametrical statistical families; take for instance the generalized Gamma distributions or mixture of lognormals. A short review can be found in Grith and Krätchemer (2011). Still, the most popular functional form of the RND is the mixtureof lognormals (MIX).

The idea originates in the Black and Scholes (1973) model. The main assumption of this model is that the price of the underlying asset of an option, S_T, follows a Geometric Brownian Motion (GBM) process. This implies that the RND function of S_T is a lognormal:

$$q(S_T) = \frac{1}{S_T \sigma \sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln S_T - \mu}{\sigma}\right)^2} \tag{1}$$

where: μ is the log scale parameter and σ is the shape parameter of the lognormal distribution. However, the lognormal distribution does not capture the negative skewness feature previously mentioned. In addition, there are many empirical studies which attest the fact that there are differences between the theoretical Black-Scholes prices and the prices that are actually observed on the market. Consider, for instance, the existence of the volatility smile. This points out that the agents acting on the market may make more complex assumptions about the price dynamics of the underlying asset of the option than the classical GBM. In this context, it seems reasonable to assume that the RND can be modeled by a mixtureof lognormal distributions. To continue with, we write q as a sum of k-component lognormal density functions:

$$q(S_T) = \sum_{i=1}^{k} \theta_i \zeta(\mu_i, \sigma_i, S_T) \tag{2}$$

where: ζ is the lognormal density function with the form exposed in equation (1) and θ_i are mixing weights which should satisfy the conditions:

$$\sum_{i=1}^{k} \theta_i = 1, \quad \theta_i > 0$$

for each i.

Melick and Thomas (1997) decided to use a three mixturecomponent density, however this results in a number of 8 parameters to be estimated: $\mu_1, \mu_2, \mu_3, \sigma_1, \sigma_2, \sigma_3, \theta_1, \theta_2$. Usually options are traded only across a relatively small number of exercise prices. Consequently, the number of parameters that can be estimated is limited. This is the reason why, see, e.g., Bahra (1997), we decide to work with a two-mixturecomponent density which has only 5 parameters to be estimated: $\mu_1, \mu_2, \sigma_1, \sigma_2, \theta$ and in the same time offers an attractive flexibility level. In this paper, we follow the latter proposal. With this assumption, the pricing formulas for call and put options can be written as:

$$C(X,\tau) = e^{-r\tau} \int_X^{\infty} (S_T - X) q(S_T) dS_T = e^{-r\tau} \int_X^{\infty} (S_T - X) \big(\theta \zeta(\mu_1, \sigma_1, S_T) + (1-\theta) \zeta(\mu_2, \sigma_2, S_T)\big) dS_T \tag{3}$$

$$P(X,\tau) = e^{-r\tau} \int_0^{X} (X - S_T) q(S_T) dS_T = e^{-r\tau} \int_0^{X} (X - S_T) \big(\theta \zeta(\mu_1, \sigma_1, S_T) + (1-\theta) \zeta(\mu_2, \sigma_2, S_T)\big) dS_T \tag{4}$$

where: X is the strike price, r is the risk free rate and τ is the time to maturity.

The optimization problem is:

$$\min_{\mu_1,\mu_2,\sigma_1,\sigma_2,\theta} \sum_{i=1}^{n} \left\{ C(X_i,\tau) - \hat{C}_i \right\}^2 + \sum_{i=1}^{n} \left\{ P(X_i,\tau) - \hat{P}_i \right\}^2 + \left\{ \theta e^{\mu_1 + \frac{1}{2}\sigma_1^2} + (1-\theta) e^{\mu_2 + \frac{1}{2}\sigma_2^2} - e^{r\tau} S_t \right\}^2 \tag{5}$$

where: t is the current time. The minimization problem reduces to minimizing the distance between the observed call and put option prices and the theoretical prices derived under the assumption that the risk-neutral distribution is a two-component mixtureof lognormals. The third term of the minimization problem uses some additional information by exploiting the fact that in the absence of arbitrage opportunities the mean of the risk-neutral function should be equal to the forward price.

For computational ease, Bahra (1997) derived the closed-form solutions of equations (3) and (4):

$$C(X,\tau)=e^{-r\tau}\left[\theta\left\{e^{\mu_1+\frac{1}{2}\sigma_1^2}\Phi(d_1)-X\Phi(d_2)\right\}+(1-\theta)\left\{e^{\mu_2+\frac{1}{2}\sigma_2^2}\Phi(d_3)-X\Phi(d_4)\right\}\right] \tag{6}$$

$$P(X,\tau)=e^{-r\tau}\left[\theta\left\{-e^{\mu_1+\frac{1}{2}\sigma_1^2}\Phi(-d_1)+X\Phi(-d_2)\right\}+(1-\theta)\left\{-e^{\mu_2+\frac{1}{2}\sigma_2^2}\Phi(-d_3)+X\Phi(-d_4)\right\}\right] \tag{7}$$

where: Φ is the cumulative distribution function of the standard normal distribution and:

$$d_1=\frac{-\ln X+\mu_1+\sigma_1^2}{\sigma_1},\qquad d_2=d_1-\sigma_1$$

$$d_3=\frac{-\ln X+\mu_2+\sigma_2^2}{\sigma_2},\qquad d_4=d_3-\sigma_2.$$

2.2 Kernel Smoothing in Implied Volatility Space

The non-parametric estimation method that will be used in this paper is related to the result of Breeden and Litzenberger (1978) which states that the risk-neutral density is proportional to the second derivative of the European call price function with respect to the strike price:

$$q(S_T)=e^{r\tau}\frac{\partial^2 C}{\partial X^2}\Big|_{X=S_T} \tag{8}$$

In practice, this derivative is very difficult to estimate because the available option data are not continuous. We can only observe option prices for a discrete number of strikes. Therefore, to construct a continuous function we need to fit a smoothing function.

Rookley (1997) estimates a smooth call price function with respect to moneyness $M_t = X/S_t$. Considering the observation that implied volatilities tend to be less volatile than the option prices, Rookley (1997) transfers call prices into implied volatility space. In the implied volatility space local polynomial smoothing is applied and the risk-neutral density is computed as a function of the implied volatility and its first and second derivative with respect to moneyness. More specifically, the model utilized is:

$$c_{it}=c(M_{it},\sigma(M_{it}))=\Phi(d_1)-\frac{e^{-r\tau}\Phi(d_2)}{M_{it}}$$

$$d_1=\frac{\ln(M_{it})+\left(r+\sigma(M_{it})^2/2\right)\tau}{\sigma(M_{it})\sqrt{\tau}} \tag{9}$$

$$d_2=d_1-\sigma(M_{it})\sqrt{\tau}$$

Based on this model, the risk-neutral density function takes the following form:

$$q(X) = e^{-r\tau} S \frac{\partial^2 c}{\partial X^2} \tag{10}$$

where:

$$\frac{\partial^2 c}{\partial X^2} = \frac{d^2 c}{dM^2}\left(\frac{M}{X}\right)^2 + 2\frac{dc}{dM}\frac{M}{X^2}.$$

We drop the indices for expositional puposes.

In turn, this depends on: $\frac{\partial \sigma(M_{it})}{\partial M_{it}}$ and $\frac{\partial^2 \sigma(M_{it})}{\partial M_{it}^2}$. The RND can be expressed in terms of the two latter terms and the implied volatility. Rookley (1997) provides complete computation. Note that in this paper we use local polynomial smoothing of degree 3 and a quartic kernel to smooth in the implied volatility space.

2.3 Evaluation of the RND Forecast Ability

Using both MIX and Rookley methods we are able to estimate the RND of the return of an option underlying at the maturity of the option. Each day we are able to estimate a different RND and, thus, we can say that this distribution is time-varying. To test whether these forecasted densities fit the true densities we take advantage of the fact that in our approach we can observe the actual realization of the option underlying. Berkowitz (2001) exploits this information to check whether the estimated densities are equal to the true densities. Of course, one can never know the true density; therefore the test uses two prior transformations so that the transformed data should follow a known distribution under the null.

The first transformation, also called the probability integral transform (PIT), is:

$$y_t = \int_{-\infty}^{S_{T,t}} q_t(u)du$$

where: $S_{T,t}$ is the realization at time $T = t + \tau$ of the density estimated at time t for future moment T . Under the assumption that the estimated q coincide with the true density and that the underlying realizations $S_{T,t}$ are independent, y_t should follow $i.i.d.U(0,1)$. The second transformation:

$$z_t = \Phi^{-1}(y_t)$$

should ensure that under the null z_t follows $i.i.d.\,N(0,1)$, where Φ is the cumulative distribution function of the standard normal distribution. The test further checks the normality of the transformed data through the first two moments by employing a likelihood ratio test:

$$LR = -2\{L(0,1,0) - L(\hat{\mu}, \hat{\sigma}, \hat{\rho})\} \tag{11}$$

where: $\hat{\mu}$ and $\hat{\sigma}$ are the mean and variance of the z_t process, $\hat{\rho}$ is the correlation coefficient of a fitted $AR(1)$ process to $(z_t - \mu)$ and L is the likelihood function of an $AR(1)$ with Gaussian error terms. The distribution of the Berkowitz test is $\chi^2(3)$.

As a comparison criterion we use the loglikehood function of the forecasted densities estimated at the actual realizations:

$$l(S_{T,1}, S_{T,2}, \ldots, S_{T,n}) = \sum_{t=1}^{n} \log \hat{q}_t(S_{T,t}) \tag{12}$$

where: n is the number of cross sections included in the sample, that is the number of pairs of estimated PDFs and their realizations.

3. Data Description

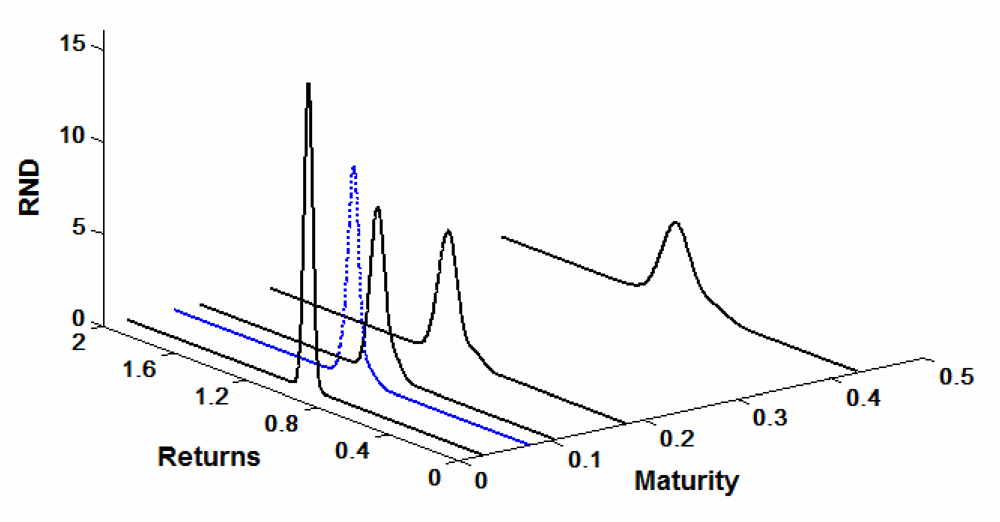
The dataset used in this paper consists of ODAX options traded on EUREX. The ODAX option contract is a European option on the DAX30 index. The dataset ranges from January 2002 to December 2011. The DAX30 daily opening prices are downloaded from Datastream for the same period of time. As a proxy for the risk-free interest rate we have used the LIBOR. Particular maturities τ of the risk-free interest rate have been obtained by linearly interpolating from the available LIBOR rates with closest maturities. Data was provided by the Research Data Center at the Collaborative Research Center 649: Economic Risk.

For each day we have estimated the risk-neutral density of the gross return of the DAX30 for all available maturities of the option contracts. The gross return of the DAX30 is defined as $S_{DAX30,T} / S_{DAX30,t}$. For the Rookley method we have only used *out of the money* options, as *in the money* options tend to be very thinly traded and in this way the effect of low trading was eliminated. However, for the MIX method we have used the entire set of data as we have observed that the method performs better in this case as compared to using only *out of the money* options. The maturities of interest were estimated by interpolating the nearest RNDs available. An exemplification is shown in Figure 1. For 11.04.2006 we were able to estimate the risk-neutral densities for maturities of 10, 38, 66 and 157 days, respectively. Therefore, to obtain a density for maturity of interest of 28 days (4 weeks) we have linearly interpolated between the maturities of 10 and 38 days.

In this paper we are interested in maturities of 2, 3, 4, 5 and 6 weeks, respectively. From the available dataset we were able to extract 20 samples of independent 4 weeks maturity RNDs and the corresponding realization of the index return. Each sample starts on a different date and estimated densities are extracted each 28 days. Therefore, a sample includes between 116 and 129 pairs of estimated risk-neutral density and realization. We have proceeded in the same way for all other maturity horizons. More details can be found in Table 1. This sampling procedure has enabled us to obtain more than 1 sample per maturity horizon. Note that most of the literature of implied option densities focuses only on densities estimated for the official expiration dates of the options. However, in this way we should only be able to obtain only one sample per maturity horizon.

Figure 1

RND on 11/04/2006 for maturities of 10, 38, 66, 157 days (continuous lines) and 28 days (dashed line)



4. Empirical Results

To begin with, for each maturity horizon and each sample available we have applied the Berkowitz test to check how the forecasted density fits the true realizations of the returns of the DAX30 index and we have calculated the likelihood. In Table 1 we display a summary of the results that we have obtained. To grasp the feeling of how these results look like, in Annex 1 we show them in detail for a 4 weeks maturity horizon. At request, detailed results can be provided by the authors for all maturity horizons under discussion.

Coming back to Table 1, in the fifth and sixth column one may see the proportion of samples in percentage where the Berkowitz test does not reject the null hypothesis at a 5% significance level for the MIX and the Rookley method, respectively; that is, that the estimated RND fits the actual realizations of the index. The seventh column provides information over which density model is preferred and in which proportion according to the likelihood criterion and irrespectively of the fact that the null hypothesis of the Berkowitz test was rejected or not.

Table 1

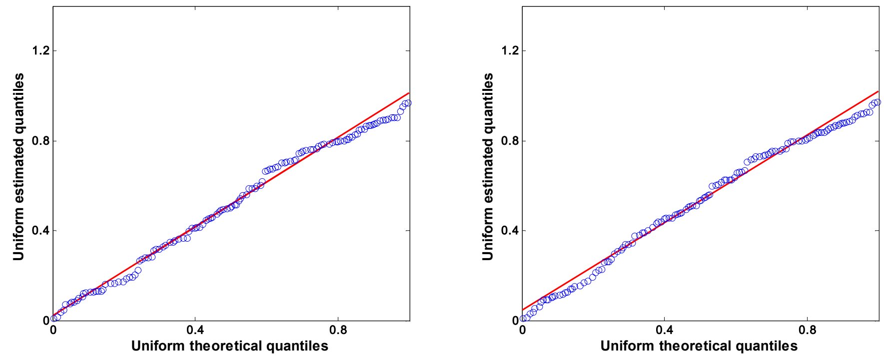
Overall information of the two estimation methodologies: MIX – mixtureof lognormals and ROOKLEY – Rookley (1997)

Maturity	Samples available	Number of cross sections		RND MIX	RND ROOKLEY	Likelihood
		Min	Max			Criterion
2 weeks	10	238	257	0	0	MIX 100%
3 weeks	15	155	170	0	0	MIX 80%
4 weeks	20	116	129	90%	100%	MIX 70%
5 weeks	25	94	104	32%	36%	ROOKLEY 60%
6 weeks	30	76	87	67%	80%	ROOKLEY 63%

For the 4 weeks maturity horizon the RND estimated by the Rookley method is not rejected in any of the samples as being a good predictor of the true density. The MIX estimate is not rejected in 90% of the samples. For 5 and 6 weeks maturity horizons the null hypothesis that the RND obtained by Rookley method fits the actual realizations is rejected in fewer cases than the mixtureof lognormals RND. Still, the RND does not perform as good as for the 4-week horizon. Surprisingly, considering the performance of the RND for wider maturity horizons, for 2 and 3 weeks maturity horizons the RND is rejected as being a good predictor of the actual distribution of the index price returns. It looks as if risk-neutral investors decide to invest only in options with longer maturity horizons, while non-risk-neutral agents invest mostly for shorter maturity horizons. Bliss and Panigirtzoglou (2004) estimated RNDs are rejected as good forecasters for all 2-6 week horizons for FTSE100 and S&P500 option data between 1992 and 2001 and 1983 and 2001, respectively. Schakleton *et al*. (2010) conclude that mixturelognormal RNDs are not reliable predictors of the PDFs of the S&P500 for 1 day horizons and 1 to 12 weeks horizons for the time period 1994 - 2010. However, using the same data set they cannot reject the Heston RND as being accurate predictors for 4-12 week horizons, which would be a similar situation to what we have encountered for our data set. Their explanation is that the Berkowitz test does not reject for longer forecast horizons due to the fact that the Berkowitz test loses power to reject when the sample size decreases. Yet, the fact that in our approach the Berkowitz test is able to reject the null hypothesis even for smaller sample sizes could be evidence that this is not the case. However, as far as we know, most of the currently available studies refer in particular to 4-week horizons. For instance, Liu *et al*. (2007) also do not reject the null hypothesis for a 4-week horizon for options written on FTSE100 between 1993 and 2002, but they affirm that a mixtureof lognormals estimates perform better than the non-parametric spline estimates proposed by Bliss and Panigirtzoglou (2002). Alonso *et al*. (2005) also cannot reject the hypothesis that RND provide accurate predictions of the pdf of future realizations of the IBEX35 for 4-week maturity between 1996 and 2003. Nevertheless, they argue that they cannot find any clear cut between the parametric mixtureof lognormals and the non-parametric spline method.

Figure 2

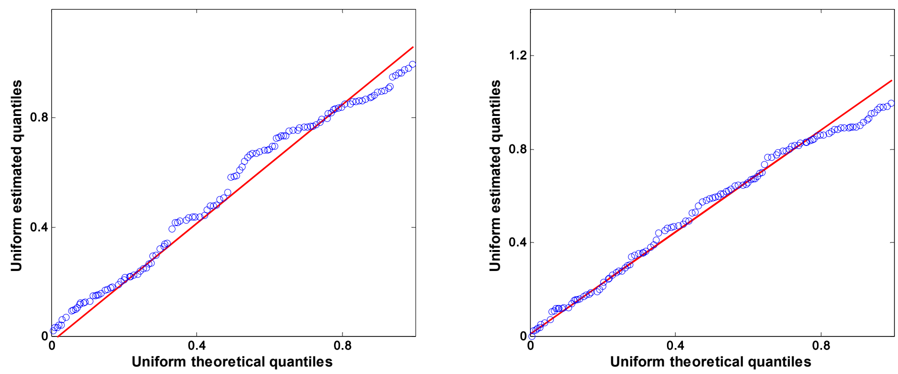
PIT for estimated MIX RND (right) and estimated ROOKLEY RND (left), 4-week maturity horizon, sample 5



Our results also do not indicate a clear preference for one method or another, but there would be some aspects worth mentioning in order to understand where the slight differences between the two methods come from. By plotting the QQ graphs of the PIT and uniform distribution, one may observe that both methods have problems with estimating the right tail, see, e.g., Figure 2. Still, there are situations when ROOKLEY RND manages to capture better the mean and the left tails, as we may see, for example, in Figure 3.

Figure 3

PIT for estimated MIX RND (right) and estimated ROOKLEY RND (left), 4-week maturity horizon, sample 1

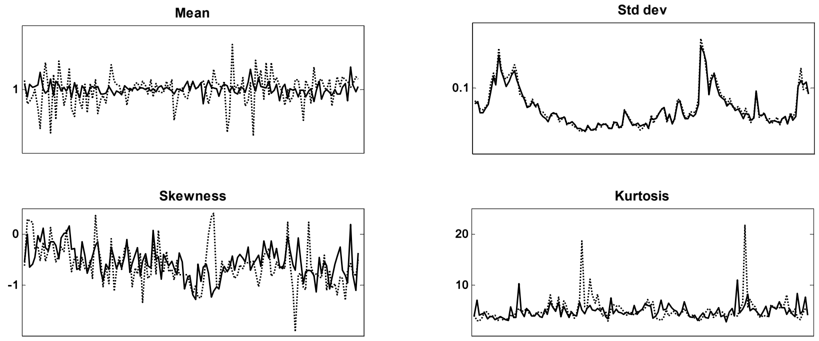


Moreover, we have plotted the first four moments of the estimated distributions, for the estimated PDFs each day, for each sample. In Figure 4 such a display is presented for sample 10, 4-week maturity horizon. While standard deviation looks similar for both methods, there are visible differences in the estimation of the mean and skewness.

With respect to kurtosis, both methods perform relatively similarly, except for some spikes that we may observe for the kurtosis characterizing the MIX method.

Figure 4

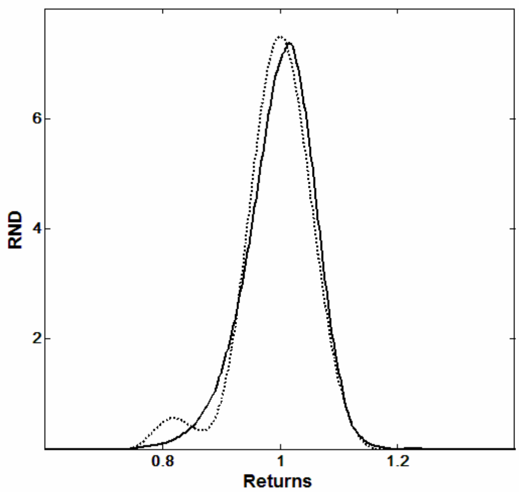
Comparison of mean, standard deviation, skewness and kurtosis for estimated MIX RND (dashed black line) and ROOKLEY RND (continous black line) at all moments of time, 4-week maturity, sample 10



For a very small number of days in the whole analyzed period, we could observe that MIX RNDs display a bimodal aspect as in Figure 5. While this is insignificant when looking at the whole period, this might matter when looking at the estimated densities on a daily basis. Bimodality of densities is often for the financial market supervisors an indicator of price manipulation on the market.

Figure 5

MIX RND (dotted line) and ROOKLEY RND (continuos line) on 15 Jan 2002, 4-week maturity



5. Conclusions

In this paper we have examined the forecasting ability of the risk-neutral density implied from options. We have looked into the performance of two different estimation methods: a two-component mixtureof lognormals and the kernel smoothing method in implied volatility space proposed by Rookley. We have used option data on the DAX30 index from 2002 to 2011 and we have checked 5 different maturity horizons: 2, 3, 4, 5 and 6 weeks. The risk-neutral density was rejected as being a good predictor of the actual realizations for the first two maturity horizons of 2 and 3 weeks. Instead, the risk-neutral density is a good predictor for the 4-week horizon densities. For the 5 and 6-week horizon densities the results do not point to a clear conclusion.

Both methods successfully manage to capture the fat tail behavior of index return distribution. The Rookley method seems to slightly overtake the mixtureof lognormals method. However, the parametric method has the advantage of producing an analytic form of the PDF, while for the Rookley method we can only obtain a non-parametric form. In addition, as for all non-parametric methods, the Rookley method does not guarantee the non-negativity and integrability to 1 properties of a density, therefore in the end they need to be corrected to comply with these properties.

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Appendix 1

Performance results of the mixtureof lognormals (MIX) and Rookley method for estimation of RND for a 4-week maturity horizon.

Sample	MIX		ROOKLEY	
	Berkowitz p-value	LogLikelihood	Berkowitz p-value	LogLikelihood
1	0.6049	172.475	0.1794	170.1298
2	0.858	172.9934	0.614	170.1287
3	0.4065	165.1328	0.6268	164.2756
4	0.3735	173.7322	0.4727	171.9853
5	0.1658	188.4577	0.2279	189.3087
6	0.0887	191.613	0.2446	184.1933
7	0.052	192.4909	0.1627	186.3782
8	0.2381	177.4052	0.3737	178.7803
9	0.0782	183.2007	0.1812	184.1706
10	0.0349	196.3674	0.1218	193.0172
11	0.0142	199.5025	0.0536	197.7336
12	0.0561	196.9855	0.0734	193.9268
13	0.2965	183.3068	0.1797	188.0735
14	0.3135	186.4479	0.5799	183.7543
15	0.5768	189.9209	0.5033	189.3052
16	0.6295	187.2835	0.8509	182.9636
17	0.7147	179.0639	0.4054	185.6748
18	0.5759	164.9511	0.5907	169.6373
19	0.7988	160.0589	0.7522	161.5087
20	0.6276	165.355	0.1998	168.803

Note that the Berkowitz test rejects the null hypothesis that the MIX RND is an accurate predictor of the true PDF of the DAX30 returns in 2 out of the 20 samples at a 5% significance level, while for the Rookley method the Berkowitz test does not reject in any of the cases. However, the estimated loglikelihood is bigger for MIX RND in 14 out of the 20 samples.

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