Abstract
The present study emphasizes the importance of human capital in economic growth. We simulate possible growth paths assuming that the Romanian economy behaves according to the hypothesis of the Uzawa-Lucas model. By calibrating the model to the Romanian economy, we are able to forecast the evolution of the Romanian GDP and the proportion of human capital which will be used in the production of goods and services. Although the population growth rate is considered to be zero, the average real GDP growth rate is around 6% due to the human capital accumulation, which improves the quality of labor.

Keywords: endogenous economic growth, human capital, two-sector economy, path simulation, Uzawa-Lucas model
JEL Classification: C15, C61, O41

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1. Introduction
Worldwide, the subject of economic growth has nowadays a central place within social and economic sciences. Theoretical and empirical research focuses on two main issues:

- To detect the mechanisms which ensure economic growth and to single out the role and the contribution of various factors to the growth process;
- To establish the microeconomic and macroeconomic policies that would ensure sustainable long-term economic growth.
Regarding the policies that ensure economic growth, an important point is to identify the factors that induce economic growth, as well as the appropriate methods by which the effect of these factors can be augmented.

One of the most important factors of economic growth is education, as an investment in what is called human capital. The importance of the education and human capital was first mentioned in the economic literature by Uzawa (1965), being reiterated later by Lucas (1988). In the recent years, numerous empirical cross-sectional studies have been employed to assess the impact of human capital on economic growth. Barro (1991) found that schooling is positively related to economic growth. The growth in per capita GDP over the period 1960 to 1990 has significantly been caused by increasing the total number of schooling years by one year. Testing the statistical relationship between education and economic growth by examining the distribution of educational expenditure to different levels of education, Gupta et al. (1999) found out that more important that total education spending is the allocation of this expenditure. Using a modified version of the Uzawa-Lucas model, Greiner, Semmler, and Gong (2005) conducted an empirical study to assess the role of education and human capital for economic growth of the USA and Europe. The results of their estimates show some nonlinear relationship between the educational effort, the growth rate of human capital and output.

The present study emphasizes the importance of human capital in economic growth. We simulate possible growth paths assuming that the Romanian economy behaves under the hypothesis of the Uzawa–Lucas model.

Namely, we consider an economy with two sectors, one producing goods and services, the other producing human capital. Because the contribution of physical capital to human capital formation is relatively reduced, the production function of the educational sector is linear in human capital. For the differential equations system obtained, we study the dynamics, the existence and determinacy of the steady state, analyzing also the balanced growth paths, and the conditions implying sustainability of growth.

By calibrating the model to the situation of the Romanian economy, we are able to forecast the evolution of the Romanian GDP and the proportion of human capital that will be used in the production of goods and services. Although, having in mind the current demographic developments, the population rate of growth is set to zero, it is possible to obtain substantial long term economic growth due to the accumulation of human capital which improves the quality of the labor.

The paper is organized as follows: in the second section we present the main equations of the model. In the third section we study the model dynamics, focusing on the balanced growth path. In the fourth section the parameters of the model are calibrated to the Romanian economy. In the fifth section we simulate the evolution of the real GDP and of the proportion in which the human capital stock is used in the production of goods and services. The final section concludes.
2. The model

In this section we present a Uzawa (1965) –Lucas (1988) type two-sector model of endogenous growth in which there are two reproducible factors of production, physical capital \( (K) \) and human capital \( (H) \). The human capital can be interpreted as the number of workers multiplied by the human capital of the typical worker. We assume here that the quantity of workers and the quality of workers are perfect substitutes in production. The two sectors will be referred to as the goods sector, and the educational sector.

The goods are produced according to a Cobb-Douglas technology:

\[
Y = A K^\alpha (uH)^{1-\alpha},
\]

where: \( Y \) is the output of the goods sector, \( A \) is a positive technology parameter, \( \alpha \) is the share of physical capital, and \( u \) is the fraction of human capital allocated to the goods sector. The production function for the education sector is:

\[
E = B (1-u) H,
\]

where: \( E \) is the formation of human capital, and \( B \) is the technological parameter for the education sector.

Goods may be either consumed \( (C) \) or added to the physical capital stock. The evolution of the stock of physical capital is thus given by

\[
\dot{K} = A K^\alpha (uH)^{1-\alpha} - \delta K - C,
\]

where: \( \delta \) is the depreciation rate of capital.

The evolution of the stock of human capital is

\[
\dot{H} = B (1-u) H - \delta H,
\]

where: without loss of generality, we assume that the depreciation rate of the human capital is equal to that of the physical capital.

The representative agent’s optimization problem is given as

\[
\max_{C, u} \int_0^\infty \frac{C(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt,
\]

subject to (1), (2), \( K(0) = K_0 > 0 \) and \( H(0) = H_0 > 0 \), where \( \rho \) is a constant subjective rate of time preference, and \( \theta \in (0, \infty) \setminus \{1\} \) is the inverse of the constant intertemporal elasticity of substitution.

The model contains two state variables, \( K \) and \( H \), and two decision variables, namely \( C \) and \( u \). Denoting by \( \mu_K \) and \( \mu_H \) the dual variables of the model, the Hamiltonian function of the system is:
Using Pontryagyn’s Maximum Principle, the first-order conditions for an internal solution of problem (P) are given by

\[ \frac{d}{dt} e^{-\rho t} - \mu_K = 0; \quad (3a) \]

\[ \frac{\dot{\mu}_H}{\mu_K} = \frac{A}{B} \left( 1 - \alpha \left( \frac{K}{uH} \right)^\alpha \right); \quad (3b) \]

\[ \frac{\dot{\mu}_K}{\mu_K} = -\alpha A \left( \frac{K}{uH} \right)^{\alpha-1} + \delta; \quad (3c) \]

\[ \frac{\dot{\mu}_H}{\mu_H} = -B + \delta \quad (3d) \]

plus the usual two transversality conditions:

\[ \lim_{t \to \infty} \mu_K(t) K(t) = 0, \quad (3e) \]

\[ \lim_{t \to \infty} \mu_H(t) H(t) = 0. \quad (3f) \]

In order to simplify the exposition, we have assumed that the constraints on the control variables \( 0 \leq u \leq 1, 0 \leq C \leq Y \) are all satisfied with strict inequality on the optimal path.

The necessary conditions (3a)-(3f) with equations (1) and (2) can be used to characterize the solution to (P). Since the optimal Hamiltonian is concave in \( \left( K, H \right) \), the necessary conditions (3a)-(3f) are also sufficient for the optimum in (P).

Although the Hamiltonian system

\[ \dot{K} = \frac{\partial \mathcal{H}}{\partial \mu_K}; \quad \dot{H} = \frac{\partial \mathcal{H}}{\partial \mu_H}; \quad \dot{\mu}_K = -\frac{\partial \mathcal{H}}{\partial K}; \quad \dot{\mu}_H = -\frac{\partial \mathcal{H}}{\partial H} \]

doesn’t have a steady state, it is straightforward to demonstrate the existence of a balanced growth path (BGP) satisfying conditions (3a)-(3f) and being characterized by a constant growth rate for \( K, H, C, \) and \( Y \).

What we want to do next is to reduce the dynamic system by one dimension, since the analytical study of a three-dimensional system is much easier, and to analyze the BGP properties.
3. Balanced Growth Path (BGP) and the reduced model

A BGP is defined as a set of functions of time \( \{ K(t), H(t), C(t), u(t) \} \) that solve the optimal control problem (P), such that \( K, H, \) and \( C \) grow at a constant rate, and \( u \) is constant.

In order to analyze the BGP, we introduce two new variables
\[
z \equiv A \left( \frac{uH}{K} \right)^{\frac{\gamma - \alpha}{\alpha}} = \frac{Y}{K} \quad \text{and} \quad \chi \equiv C \left( \frac{H}{K} \right)^{\frac{\gamma}{\alpha}}.
\]

Using (1) and (2) the growth rates of the two types of capital are given by
\[
\gamma_K \equiv \frac{\dot{K}}{K} = z - \alpha - \gamma - \delta,
\]
\[
\gamma_H \equiv \frac{\dot{H}}{H} = B(1-u) - \delta.
\]

It is straightforward to show that the fraction of human capital used in the production sector has the following dynamics:
\[
\gamma_u \equiv \frac{\dot{u}}{u} = B \frac{1-\alpha}{\alpha} + Bu - \chi.
\]

The dynamics of \( z \) is given by:
\[
\gamma_z \equiv \frac{\dot{z}}{z} = \gamma_y - \gamma_K = (\alpha - 1)z + \frac{(1-\alpha)}{\alpha} B.
\]

The growth rate of consumption is:
\[
\gamma_c \equiv \frac{\dot{C}}{C} = \frac{1}{\theta} \left( \alpha \cdot z - \delta - \rho \right).
\]

The dynamics of \( \chi \) is given by:
\[
\gamma_{\chi} \equiv \frac{\dot{\chi}}{\chi} = \gamma_c - \gamma_K = \left( \frac{\alpha - \theta}{\theta} \right) z + \frac{1}{\theta} \left[ \delta(1-\theta) + \rho \right].
\]

The dynamics of the model is completely determined by the differential equations (6), (7), and (9), with the initial conditions given by \( z(0) = z_0, \chi(0) = \chi_0, u(0) = u_0 \).

Using matrix notation, the system can be written as:
The existence and the uniqueness of the steady state \( (z^*, \chi^*, u^*) \) are assured by the non-singularity of the matrix \( M \). The steady state point lies at the intersection of the \( \{ z = 0, \chi = 0, u = 0 \} \) loci:

\[
\begin{align*}
\alpha - 1 & \quad 0 & 0 \\
\alpha \theta - 1 & \quad 1 & 0 \\
0 & \quad -1 & B \\
\end{align*}
\]

\[
M \begin{bmatrix} z \\ \chi \\ u \end{bmatrix} = \begin{bmatrix} \frac{(\alpha - 1)B}{\alpha} \\ \frac{1}{\theta}[\delta(1-\theta)+\rho] \\ \frac{(\alpha - 1)B}{\alpha} \end{bmatrix}.
\] (10)

The growth rate on the balanced growth path of the economy is

\[
\gamma_Y = \gamma_K = \gamma_H = \gamma_C = (B - \rho - \delta)/\theta.
\] (12)

The sustainability of the long run economic growth requires that \( \gamma_Y^* > 0 \), while the transversality condition implies \( \gamma_Y^* < B - \delta \), yielding the following restriction on the parameters:

\[
0 < B - (\rho + \delta) < \theta (B - \delta).
\] (13)

The constraint (13) is sufficient for \( z^* > 0 \) and \( \chi^* > 0 \), being also necessary and sufficient for \( u^* \in (0, 1) \), ensuring that the steady state is well defined.

The transitional dynamics of the Uzawa-Lucas model can be studied by the “time elimination” method (Barro and Sala-i-Martin, 1995).

The eigenvalues of the matrix \( M \) from (10) are \( \{ 1, B, \alpha - 1 \} \). Since \( M \) has two positive eigenvalues and a negative one \( (\alpha - 1) \) the system is characterized by saddle path dynamics. The speed of convergence during transitional dynamics is determined by the magnitude of the negative eigenvalue.

Time elimination in (10) yields the following differential system for the saddle paths of \( \ddot{z}(z), \ddot{u}(z) \):

\[
\begin{align*}
\frac{d\chi}{dz} &= \chi \left[ \frac{\alpha - \theta}{\theta(\alpha - 1)} + \frac{1}{\alpha - 1} \frac{\chi - \chi^*}{z - z^*} \right] \\
\frac{du}{dz} &= u \left( B(u - u^*) \right) - \left( \frac{\chi - \chi^*}{z - z^*} \right) \\
\end{align*}
\]

\] (14)
with the condition \( \hat{x}(z^*) = \chi^* \), \( \bar{u}(z^*) = u^* \). The phase diagram of the model is given in Figure 1.

**Figure 1**

Phase diagram of the Uzawa-Lucas model

- **a. The \((z, \chi)\) space**
  - \( \dot{z} = 0 \):
  - \( \dot{\chi} = 0 \): arrows

- **b. The \((\chi, u)\) space**
  - \( \dot{\chi} = 0 \):
  - \( \dot{u} = 0 \): arrows

The \( \dot{z} = 0 \) locus is a vertical line passing through \( z^* \), while the \( \dot{\chi} = 0 \) locus is given by the linear equation \( \chi = \chi^* + \frac{1}{\theta}(\theta - \alpha)(z - z^*) \). To analyze the evolution of \( u \) it is convenient to use the coordinate system \((\chi, u)\). Using these coordinates, the \( \dot{\chi} = 0 \) locus is a vertical line through \( \chi^* \), and the \( \dot{u} = 0 \) locus is given by \( u = u^* + \frac{1}{B}(\chi - \chi^*) \).

**4. Model calibration**

In this section we present the methodology employed for the calibration of the parameters to the Romanian economy.

The set of parameters which need to be calibrated is \( \{\theta, \rho, A, B, \alpha, \delta, z_0\} \). The parameter modeling the preferences of the households was selected according to similar studies such as Greiner (2007), Greiner, Semmler and Gong (2004), as well as Greiner and Semmler (2000): \( \rho = 0.01 \). Motivated by the recent demographic developments, we set the population rate of growth equal to zero. In this way our results reflect exclusively the qualitative effect of labor on economic growth. Following Denis et al. (2006), we assume that the output elasticity with respect to physical capital is \( \alpha = 0.37 \). The depreciation rate of capital is set to \( \delta = 5\% \). The technology parameter \( A \) is set to 0.1.
The technology parameter $B$, the parameter $\theta$ of the utility function, and $z_0$ were calibrated such that to minimize a squared error function penalizing the deviations of the simulated GDP from the actual GDP, as well as the deviations of the simulated values for the fraction of human capital used in the production sector from the actual ones. The minimization was performed for the period 2000:Q1-2005:Q4. Actual GDP values were seasonally adjusted. The data for real GDP are from the National Institute of Statistics. The actual values for the fraction of human capital used in the production sector were computed using the methodology described in Gong, Greiner and Semmler (2002) using data from Eurostat.

The minimum of the squared error function is obtained for $B = 0.051$, $\theta = 1.932$ and $z_0 = 0.198$. The calibration surfaces for parameters $B$, $\theta$ and $z_0$ are displayed in figures 2, 3 and 4, respectively.
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Squared error function in \((z_0, \theta)\) coordinates

The in-sample (2000:Q1-2005:Q4) and out-of-sample (2006:Q1 2007:Q4) simulated and actual GDP values are presented in Figure 5.

Figure 5

Simulated and actual GDP

The in-sample (2000:Q1-2005:Q4) and out-of-sample (2006:Q1 2007:Q4) simulated and actual values for the fraction of human capital used in the production sector are presented in Figure 6.

Figure 6

Simulated and actual fraction of human capital used in the production sector
The values obtained for the model parameters in the calibration process are in line with similar studies, both for Romania (Albu, 2006; Dobrescu, 2006; Caraiani, 2008) and for other economies (Gong, Greiner and Semmler, 2002). Figures 5 and 6 indicate that the calibrated model provides a good approximation for the evolution of the Romanian economy for the period 2000:Q1–2007:Q4.

5. Simulation results

In this section we use the calibrated model to obtain the growth rate on the balanced growth path, as well as the transitional dynamics for the Romanian economy. The analysis is focused on the evolution of real GDP and of the fraction of human capital used in the production sector.

The simulation process consists of the following steps:

1. determining the $\dot{u} = 0$, $\dot{\chi} = 0$, and $\dot{z} = 0$ loci;
2. computing the steady state values for $u$, $\chi$, and $z$, as well as of the growth rate on the balanced growth path;
3. obtaining the stable arm of the saddle path by numerically solving the system of differential equations (14); the stable arm consists of the functions $\tilde{u}(z)$ and $\tilde{\chi}(z)$, characterized by $\tilde{\chi}(z^*) = \chi^*$, $\tilde{u}(z^*) = u^*$.
4. obtaining consistent initial values for the variables $u$, $\chi$, and $z$; since the system exhibits saddle path dynamics, there is an unique combination of initial
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values \( \left( u_0, \chi_0, z_0 \right) \) ensuring convergence to the balanced growth path:

\[ u_0 = \bar{u} \left( z_0 \right), \quad \chi_0 = \bar{\chi} \left( z_0 \right), \] where \( z_0 \) was calibrated in the previous section.

5. solving numerically the system (10) with initial conditions from the previous step and obtaining \( u, \chi \) and \( z \) as functions of time;

6. forecasting the evolution of the real GDP, and of the fraction of the human capital used in the production sector.

The simulations performed using the calibrated model show that on the long run the annual growth rate is 6% and the human capital will be used in proportion of 46.6% in the production sector. The forecasted evolution of the Romanian real GDP for the period 2008-2020 is presented in Table 1.

<table>
<thead>
<tr>
<th>Year</th>
<th>GDP (mln. RON, prices 2000)</th>
<th>Growth rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008</td>
<td>127158.5</td>
<td>5.98</td>
</tr>
<tr>
<td>2009</td>
<td>134797.6</td>
<td>6.01</td>
</tr>
<tr>
<td>2010</td>
<td>142922.8</td>
<td>6.03</td>
</tr>
<tr>
<td>2011</td>
<td>151557.6</td>
<td>6.04</td>
</tr>
<tr>
<td>2012</td>
<td>160727.9</td>
<td>6.05</td>
</tr>
<tr>
<td>2013</td>
<td>170462.1</td>
<td>6.06</td>
</tr>
<tr>
<td>2014</td>
<td>180790.4</td>
<td>6.06</td>
</tr>
<tr>
<td>2015</td>
<td>191743.7</td>
<td>6.06</td>
</tr>
<tr>
<td>2016</td>
<td>203354.2</td>
<td>6.06</td>
</tr>
<tr>
<td>2017</td>
<td>215652.4</td>
<td>6.05</td>
</tr>
<tr>
<td>2018</td>
<td>228665.0</td>
<td>6.03</td>
</tr>
<tr>
<td>2019</td>
<td>242413.1</td>
<td>6.01</td>
</tr>
<tr>
<td>2020</td>
<td>256982.1</td>
<td>6.01</td>
</tr>
</tbody>
</table>

Although we considered the population growth rate to be zero, the average real GDP growth rate is around 6% due to the human capital accumulation, which improves the quality of labor.

The path of the human capital fraction used in the production sector is displayed in Figure 7.
6. Concluding remarks

This study employs the Uzawa-Lucas endogenous growth model to emphasize the role of human capital in economic growth in the case of the Romanian economy. The Uzawa-Lucas model is a good approximation for economies over a certain time period when education rises, implying that the economies are on the transition path to the long run steady state.

The model is calibrated by minimizing the distance between the simulated and actual paths for real GDP and fraction of human capital used in the production sector. The calibrated model provides a good approximation for the evolution of the Romanian economy both in-sample (2000:Q1-2005:Q4) and out-of-sample (2006:Q1-2007:Q4). It is important to mention that, in order to reflect exclusively the qualitative effect of labor on economic growth, the population growth rate is set to zero.

The simulations performed for the period 2008-2020 using the calibrated model show that on the long run the real GDP annual growth rate is about 6%, which is consistent with the results of similar studies using other methods (Caraiani, 2008; Pauna, Ghizdeanu, Scutaru et al., 2008). The results also indicate that on the long run the human capital will be used in proportion of 46.6% in the production sector. The simulated transitional path is similar to the actual one for the period 2000:Q1-2007:Q4, computed by the methodology in Gong, Greiner and Semmler (2002).

Given the importance of the sustainable development process, further research should also deal with the impact on the Romanian economy of other growth determinants, such as fiscal and commercial policy, public capital, R&D incentives, and non-renewable resources.
References


