7. WHY FUEL POVERTY?

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Abstract

The National Health Service (NHS) in UK prescribes 21⁰C in the living room and 18⁰C in other rooms for all households but for more than 7.7 percent households cost of heating is above 10 percent of their income and they suffered from coldness related diseases and are in fuel poverty. Raising growth rates of income of vulnerable households, stabilising prices of fuel products and improving fuel efficiency of houses are measure to eliminate fuel poverty and to reduce the excess deaths caused by this. This paper shows how the basic needs of fuel demand can be modelled using the Stone-Geary preferences and how the growth of the economy is related to those preferences and technology of firms over time is analysed with Bellman-Sargent dynamic programming models. Demand and supply analysis of this kind should complement DTI's methodology on fuel poverty for policy analysis.

Keywords: Fuel poverty, Stone Geary Preference, dynamic programming

JEL Classification*:* D01, D12, D18

1. Introduction

Winter season is very cold in England. Households in low income categories cannot heat their homes or working places to maintain the temperature inside their premises up to 21^0C in the living room and 18^0C in other rooms as prescribed by the National Health Services (NHS). The DTI (2006) determines whether a household is in fuel poverty based on whether its expenses for heating, lighting and cooking is more than 10 percent of its income. By this criterion more than 163,000 households in the Yorkshire and Humber (7.7% of the households) were found to be in fuel poverty in 2006; some 123,000 of them being in vulnerable positions. Coldness has caused illnesses and diseases such as asthma, chronic obstructive pulmonary disease, blood pressure and heart attack, arthritis, loss of strengths of fingers and mental retardation. It ultimately resulted in excess death of more than 3200 individuals annually in Yorkshire alone. It has caused social isolation of elderly and created obstacles in smooth education process of younger pupils. Government has become increasingly concerned on this recently though Rowntree (1902) had documented consequences

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of such poverty phenomena systematically for longer than a century (see also Townsend (1979)).

Rising fuel prices, lower growth rates of income of vulnerable households, rising trend in dependency ratios, fuel inefficient structure of houses and new patterns of fuel intensive consumption remain major causes of fuel poverty. One percent increase in fuel price pushes 50,000 households into fuel poverty (YGPHO (2006)). Most of these are of 60 years of age or older or families with children living in dwellings without central heating or in older and larger properties.

DTI (2006) reported that Winter fuel payments had lifted many pensioners out of fuel poverty. Home improvement measures including draught proofing, cavity and loft insulations, installation of gas central heating, oil fired central heating, CHP community heating, boiler replacements, solid wall external and electric storage are recommended as a strategy to end fuel poverty by 2016.

This paper provides analytical solutions of this problem illustrating how Stone-Geary preferences and Bellman-Sargent type dynamic programming models can by applied to think about this issue drawing on empirical facts based on Food and Expenditure Survey (FES(2003)), population projections and other statistics obtained from the Office of National Statistics. Stone-Geary preference is appropriate to model the minimum consumption need of fuel for any point of time and dynamic programming model shows how preferences regarding current the future consumptions ultimately determine the trajectory of capital, output and living standards over time.

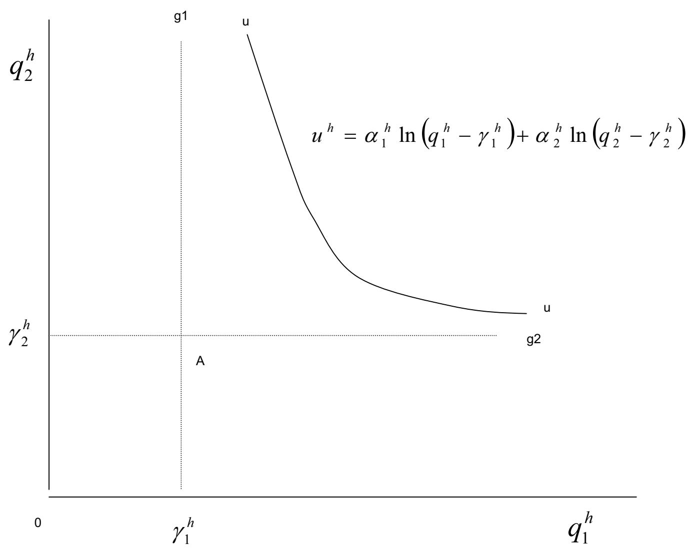
2. Stone Geary Preferences for Analysing Fuel Poverty

Stone-Geary (1949-50) preference, as presented in Figure 1, is useful for analysing household demand with minimum heating needs and fuel poverty. Let q_1^h represent the demand for fuel of household *h* and γ_1^h be the minimum required according to the health standards. Similarly let all other items such as housing, clothing, transportation, communication, education, health, recreation and all other goods needed to maintain a reasonable standards of living be given by q_2^h with the basic minimum standard set at γ_2^h. A certain household *h* need to be at least at point A in order to have basic minimum need of both γ_1^h and γ_2^h. The vertical line $\gamma_1^h g_1$ and horizontal line $\gamma_2^h g_2$ show respectively the minimum amount of good 1 and 2 required for a healthy life; the indifference curve uu shows a given level of utility from consumption of combinations of q_1^h and q_2^h. Consumption level below point A condemns one into poverty.

I follow Stone (1954) and Handerson and Quandt (1980) in deriving fuel demands according to the Stone-Geary preferences. Let the utility $u^h = \alpha_1^h \ln\left(q_1^h - \gamma_1^h\right) + \alpha_2^h \ln\left(q_2^h - \gamma_2^h\right)$ monotonically be transformed to $u^{h'} = B_1^h \ln\left(q_1^h - \gamma_1^h\right) + B_2^h \ln\left(q_2^h - \gamma_2^h\right)$ with $B_1^h = \frac{\alpha_1^h}{\alpha_1^h + \alpha_2^h}$ and $B_2^h = \frac{\alpha_2^h}{\alpha_1^h + \alpha_2^h}$

Figure 1

Stone Geary Preferences for Fuel and Other Goods



where α_1^h and α_2^h denote share of spending on good 1 and 2 respectively. Then the consumer's optimisation problem can be stated as:

$$\text{Max} \quad u^{h'} = B_1^h \ln\left(q_1^h - \gamma_1^h\right) + B_2^h \ln\left(q_2^h - \gamma_2^h\right) \tag{1}$$

Subject to

$$y^h = p_1 q_1^h + p_2 q_2^h \tag{2}$$

The Lagrangian constrained optimisation function for this problem becomes:

$$L\left(q_1^h, q_2^h, \lambda^h\right) = B_1^h \ln\left(q_1^h - \gamma_1^h\right) + B_2^h \ln\left(q_2^h - \gamma_2^h\right) + \lambda^h\left(y^h - p_1 q_1^h - p_2 q_2^h\right) \tag{3}$$

Optimal conditions for choice of three variables are:

$$\frac{\partial L\left(q_1^h, q_2^h, \lambda^h\right)}{\partial q_1^h} = \frac{B_1^h}{\left(q_1^h - \gamma_1^h\right)} - \lambda^h p_1 = 0 \tag{4}$$

$$\frac{\partial L\left(q_1^h, q_2^h, \lambda^h\right)}{\partial q_2^h} = \frac{B_2^h}{\left(q_2^h - \gamma_2^h\right)} - \lambda^h p_2 = 0 \tag{5}$$

$$\frac{\partial L\left(q_1^h, q_2^h, \lambda^h\right)}{\partial \lambda^h} = y^h - p_1 q_1^h - p_2 q_2^h = 0 \tag{6}$$

Rearrange (4) to get

$$p_1 q_1^h = p_1 \gamma_1^h + \frac{B_1^h}{\lambda^h} \tag{7}$$

Similarly rearrange (5) to get

$$p_2 q_2^h = p_2 \gamma_2^h + \frac{B_2^h}{\lambda^h} \tag{8}$$

Now using (7) and (8) in (6) to get

$$y^h - p_1 \gamma_1^h - \frac{B_1^h}{\lambda^h} - p_2 \gamma_2^h - \frac{B_2^h}{\lambda^h} = 0$$

$$\frac{B_1^h + B_2^h}{\lambda^h} = y^h - p_1 \gamma_1^h - p_2 \gamma_2^h \text{ or } \frac{1}{\lambda^h} = y^h - p_1 \gamma_1^h - p_2 \gamma_2^h \tag{9}$$

Put (9) into (7) $p_1 q_1^h = p_1 \gamma_1^h + B_1^h \left(y^h - p_1 \gamma_1^h - p_2 \gamma_2^h \right)$ to get

$$q_1^h = \gamma_1^h + \frac{B_1^h}{p_1} \left(y^h - p_1 \gamma_1^h - p_2 \gamma_2^h \right) \tag{10}$$

Similarly put (9) into (8) $p_2 q_2^h = p_2 \gamma_2^h + B_2^h \left(y^h - p_1 \gamma_1^h - p_2 \gamma_2^h \right)$ to get

$$q_2^h = \gamma_2^h + \frac{B_2^h}{p_2} \left(y^h - p_1 \gamma_1^h - p_2 \gamma_2^h \right) \tag{11}$$

From demand functions (10) and (11) one can determine whether a household *h* is in fuel poverty comparing whether q_1^h is greater or less than the minimum required γ_1^h which depends on its income relative to amount required for the minimum need, $\left(y^h - p_1 \gamma_1^h - p_2 \gamma_2^h \right)$. Three different scenarios emerge from this analytical solution:

1) Household *h* is below the point A in above diagram if $\left(y^h - p_1 \gamma_1^h - p_2 \gamma_2^h \right) < 0$; this household faces fuel poverty and is in vulnerable situation.
2) The household's budget allows it to meet its minimum fuel needs if $\left(y^h - p_1 \gamma_1^h - p_2 \gamma_2^h \right) = 0$. Such household barely manages to be out of fuel poverty and is at point A in the above diagram.
3) Household is above the basic need point A if $\left(y^h - p_1 \gamma_1^h - p_2 \gamma_2^h \right) > 0$. Ideally the UK government aims to bring every household to this level by 2016.

Stagnant level of income y^h and steadily rising prices of fuel (p_1) and other products (p_2) can push a household from position (3) to position (2) or even to position (1). When income of a household does not rise in proportion to increase in prices, there is a greater probability that it will fall into the fuel poverty trap. There are many reasons why the level of income (y^h) does not increase as the expenditure needed to meet the minimum requirement ($p_1 \gamma_1^h + p_2 \gamma_2^h$) rises.

The FES (2003) reveals significant differences in income and expenditure patterns of rich and poor households. On average each household had 2.4 individuals in the last 25 years. The wages and salaries was their major sources of income. Their weekly income had grown annually only by 1.67 percent on average in last 20 years; rising very differently for different categories of households. In year 2003 the weekly income of a household in the richest decile was £1085 - nine times higher than £123 of 2nd poorest decile. Higher weekly income allows the richest decile to spend 6.5 times more on average, 25 times more in catered food, 17 times more in package holidays, 13 times more in cinema, theatre and recreation, 10 times more in transportation, personal effects including furniture, clothes, accessories, magazines and similar products than of households in the poorest decile with only around 45 percent of the average spending. The fuel products were becoming more expensive to households up to the fourth decile as were the housing services. Low income households spent about £8 pounds per week (6.5 percent of their income) in gas, electricity or other fuel products whereas the richest ones spent about £17 (about 1.5 percent of their weekly income).

Fuel poverty may rise in coming years for a number of factors - 1) dependency ratio rising with more elderly people in population that is growing annually rate of 0.51 percent compared to 0.2 percent growth rate for the working age population (16-64); 2) fuel prices are more likely to rise after depletion of the North Sea oil; 3) DTI (2003) programme on improvement of 2 million houses to lift 3.3 million people out of fuel poverty is likely to be inadequate to meet growing demand for fuel efficient housing; 4) rate of technical advancement and formation of human capital is not adequate to spur growth because of unpopularity of science and technology subjects among young pupils in recent years in sharp contrast to the historical trends in UK. This will create gaps in innovations and productivity. Head count ratio used in the DTI methodology of fuel poverty measurement thus need to be complemented by proper considerations of these dynamic factors briefly illustrated in the dynamic programming model in the next section. This can be expanded numerically to make it more realistic using dynamic general equilibrium models.

3. Dynamic programming model for analysing fuel poverty

Fuel poverty is a dynamic issue; households are in and out of it depending on dynamics of income and expenditure which ultimately depend on the dynamics of the economy as represented in a problem of a representative household in a dynamic economy:

$$Max \quad U = \sum_{t}^{\infty} \beta^t \ln C_t \qquad 0 < \beta < 1 \tag{12}$$

subject to

$$K_{t+1} + C_t = AK_t^{\alpha} \qquad 0 < \alpha < 1 \tag{13}$$

where the future utility is weighted less than the current utility $(\ln C_t)$ as the discount factor is between zero and one, $0 < \beta < 1$. In the context of fuel poverty C_t is composite of q_1^h and q_2^h, quantities of fuel and non fuel products. Similarly the output $Y_t = AK_t^\alpha$ also is composite of these two products, $q_{1,t}$ and $q_{2,t}$. The market clearing condition implies that $\sum_h q_{1,t}^h = q_{1,t}$ and $\sum_h q_{2,t}^h = q_{2,t}$. Capital stock is similarly divided in producing fuel and non-fuel products, $K_t = K_{1,t} + K_{2,t}$; housing and non-housing capital stocks. Bellman (1957) and Sargent (1987) technique can be used to solve for state and co-state variables iteratively from the following value function.

$$V_1(K) = \max_k \{\ln C + \beta \ln(V_0(K'))\} \tag{14}$$

When the capital stock at the terminal period is made zero, $C_t + K' = AK^\alpha$, $C_t = AK^\alpha$, all output is consumed and household utility is given by:

$$V_1(K) = \ln C = \ln(AK^\alpha) = \ln A + \alpha \ln K \tag{15}$$

This solution can then be used in the policy function for the next period

$$V_2(K) = \ln C + \beta \ln(V_1(K')) = \ln C + \beta(\ln A + \alpha \ln K) = \ln(AK^\alpha - K') + \beta(\ln A + \alpha \ln K)$$

Then again the consumer has to decide how much to save for the next period

$$V_{\underset{K'}{2}}(K) = \ln(AK^\alpha - K') + \beta(\ln A + \alpha \ln K) \tag{16}$$

Now the optimal value of K', the capital to be saved for the next period in the second last period can be obtained using the first order conditions:

$$\frac{\partial V_2(K)}{\partial K} = -\frac{1}{AK^\alpha - K'} + \frac{\beta\alpha}{K'} = 0 \qquad \frac{1}{AK^\alpha - K'} = \frac{\beta\alpha}{K'}$$

$$K' = \beta\alpha(AK^\alpha - K') \qquad \text{or } K'(1+\beta\alpha) = \beta\alpha AK^\alpha \qquad K' = \frac{\beta\alpha}{(1+\beta\alpha)} AK^\alpha$$

Again consumption is total output minus the savings for the next period

$$C = AK^\alpha - \frac{\beta\alpha}{(1+\beta\alpha)} AK^\alpha \qquad \text{or } C = \frac{1}{(1+\beta\alpha)} AK^\alpha$$

$$V_2(K) = \ln C + \beta V_1(AK^\alpha) = \ln\left[\frac{1}{(1+\beta\alpha)} AK^\alpha\right] + \beta(\ln A + \alpha \ln K')$$

This now contains only one state variable, the capital stock.

$$V_2(K) = \ln\left[\frac{1}{(1+\beta\alpha)} AK^\alpha\right] + \beta \ln A + \beta\alpha \ln\left(\frac{\beta\alpha}{(1+\beta\alpha)} AK^\alpha\right)$$

$$V_2(K') = \ln\left[\frac{1}{(1+\beta\alpha)}A\right] + \beta \ln A + \beta\alpha \ln\left(\frac{\beta\alpha}{(1+\beta\alpha)}A\right) + \alpha(1+\alpha\beta)\ln K' \quad (17)$$

In a similar fashion consider the problem in the third last period given $V_2(K')$ in (17).

$$V_3(K) = \ln C + \beta V_2(K') \quad (18)$$

Using the market clearing condition $C_t = AK^{\alpha} - K'$ and above solution becomes:

$$\underset{\max k}{V_3}(K) = \ln(AK^{\alpha} - K') + \beta(\alpha(1+\alpha\beta)\ln K')$$

The optimal first order conditions on control (consumption) and state (capital stock) variables. $\underset{\max k}{\frac{\partial V_3(K)}{\partial K}} = -\frac{1}{AK^{\alpha} - K'} + \frac{\beta\alpha(1+\alpha\beta)}{K'} = 0$ or

$$\frac{1}{AK^{\alpha} - K'} = \frac{\beta\alpha(1+\alpha\beta)}{K'}$$

$$K' = \beta\alpha(1+\alpha\beta)(AK^{\alpha} - K')$$

$$K' = \frac{(\beta\alpha + \alpha^2\beta^2)}{(1+\beta\alpha+\alpha^2\beta^2)}AK^{\alpha}$$

$$C = (AK^{\alpha} - K') = \left[AK^{\alpha} - \frac{(\beta\alpha+\alpha^2\beta^2)}{(1+\beta\alpha+\alpha^2\beta^2)}AK^{\alpha}\right]$$

$$C = \frac{1}{(1+\beta\alpha+\alpha^2\beta^2)}AK^{\alpha}$$

Combining solutions for periods two and three return function for period 3 is

$$V_3(K') = \ln\left[\frac{1}{(1+\beta\alpha+\alpha^2\beta^2)}AK^{\alpha}\right] + \beta\left[\ln\left[\frac{1}{(1+\beta\alpha)}A\right] + \beta\ln A + \beta\alpha\ln\left(\frac{\beta\alpha}{(1+\beta\alpha)}A\right) + \alpha(1+\alpha\beta)\ln K'\right]$$

Upon further simplification by collecting terms involving capital stock it becomes

$$V_3(K') = \beta\ln\left[\frac{A}{(1+\beta\alpha)}\right] + \beta^2\ln A + \beta^2\alpha\ln\left(\frac{\beta\alpha A}{(1+\beta\alpha)}\right) + \ln\left(\frac{A}{(1+\beta\alpha+\alpha^2\beta^2)}\right) + \beta\alpha(1+\alpha\beta)\ln\left[\frac{(\beta\alpha+\alpha^2\beta^2)A}{(1+\beta\alpha+\alpha^2\beta^2)}\right] \quad (19)$$
$$+\alpha(1+\beta\alpha+\alpha^2\beta^2)\ln K'$$

The problems in the fourth last period can be solved taking the optimal solution of the third last period problems $V_3(K')$ as given above as following:

$$V_4(K) = \ln C + \beta V_3(K') = \ln(AK^{\alpha} - K') + \alpha(1+\beta\alpha+\alpha^2\beta^2)\ln K' \quad (20)$$

Again using the first order conditions

$$\frac{1}{(AK^{\alpha} - K')} = \frac{\alpha\beta(1+\beta\alpha+\alpha^2\beta^2)}{K'}$$

$$K' = \frac{\left(\beta\alpha + \alpha^2\beta^2 + \alpha^3\beta^3\right)}{1 + \alpha\beta + \alpha^2\beta^2 + \alpha^3\beta^3} AK^\alpha$$

$$C = AK^\alpha - \frac{\left(\beta\alpha + \alpha^2\beta^2 + \alpha^3\beta^3\right)}{1 + \alpha\beta + \alpha^2\beta^2 + \alpha^3\beta^3} AK^\alpha \qquad C = \frac{1}{1 + \alpha\beta + \alpha^2\beta^2 + \alpha^3\beta^3} AK^\alpha$$

Using the optimal solution for the next period the value function of the 4th last period is:

$$V_4(K') = \ln\left[\frac{1}{1 + \alpha\beta + \alpha^2\beta^2 + \alpha^3\beta^3} AK^\alpha\right] + \beta \left[\begin{array}{l} \beta \ln\left[\frac{A}{(1+\beta\alpha)}\right] + \beta^2 \ln A + \beta^2\alpha \ln\left(\frac{\beta\alpha A}{(1+\beta\alpha)}\right) \\ + \ln\left(\frac{A}{(1+\beta\alpha+\alpha^2\beta^2)}\right) + \beta\alpha(1+\alpha\beta)\ln\left[\frac{(\beta\alpha+\alpha^2\beta^2)A}{(1+\beta\alpha+\alpha^2\beta^2)}\right] \\ + \alpha(1+\beta\alpha+\alpha^2\beta^2)\ln\left[\frac{\left(\beta\alpha+\alpha^2\beta^2+\alpha^3\beta^3\right)}{1+\alpha\beta+\alpha^2\beta^2+\alpha^3\beta^3} AK^\alpha\right] \end{array}\right]$$

Optimal capital accumulation after 4th iteration becomes:

$$\begin{aligned} V_4(K') = {} & \ln\left[\frac{1}{1+\alpha\beta+\alpha^2\beta^2+\alpha^3\beta^3} A\right] + \beta\ln\left(\frac{A}{(1+\beta\alpha+\alpha^2\beta^2)}\right) + \beta^2\left[\frac{\ln A}{(1+\beta\alpha)}\right] + \beta^3 \ln A \\ & + \beta\alpha(1+\beta\alpha+\alpha^2\beta^2)\ln\left[\frac{\left(\beta\alpha+\alpha^2\beta^2+\alpha^3\beta^3\right)\alpha\beta A}{1+\alpha\beta+\alpha^2\beta^2+\alpha^3\beta^3}\right] + \beta\left\{\beta\alpha(1+\alpha\beta)\ln\left[\frac{(\beta\alpha+\alpha^2\beta^2)A}{(1+\beta\alpha+\alpha^2\beta^2)}\right]\right\} + \beta^2\left\{\beta\alpha\ln\left[\frac{\alpha\beta A}{(1+\beta\alpha)}\right]\right\} \\ & + \alpha\left[1+\beta\alpha\left(1+\beta\alpha+\alpha^2\beta^2+\alpha^3\beta^3\right)\right]\ln K' \end{aligned} \quad (21)$$

Thus in a dynamic economy accumulation consistent to removal of fuel poverty should depend on preferences of households, their discount factors, technology of firms and initial and terminal conditions. Economy may experience various trajectories of growth paths depending on configurations of these parameters. One numerical example of a dynamic applied general equilibrium model for the Humberside economy is in Bhattarai (2007); in stochastic settings these solutions can be approximated by more robust estimation.

Considering above two model predictions one could easily argue that solving the fuel poverty requires not only consideration of fulfilment of the basic minimum fuel need but also a growing economy where incomes of households' keep rising so that they are able to afford adequate amounts of such necessary commodity. This requires right preferences for current versus future consumption and efficiency of capital accumulation to maximise the welfare of household for long time to come.

4. Conclusion

A household is considered to be in fuel poverty when the cost of heating room up to 21^0C/18^0C is above 10 percent of their household income.This is issue of fulfilment of basic minimum heating need as well as the dynamic and vibrant economy. Major reasons for fuel poverty are lower growth rates of income of households in low income groups, rising prices of fuel products, fuel inefficient structure of houses and households and patterns of their consumption. Solving this problem requires consideration of dynamic factors in addition to static factors outlined in DTI (2006). Paper show how the basic need demand side of fuel poverty could be analysed using the Stone-Geary preference and how this should be related to a dynamically efficient and optimal economy as shown by the analytical solutions of the dynamic programming models of Bellman (1957) and Sargent (1987). Empirical evidences for the study are drawn on the Family Expenditure Survey of England and Yorkshire

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