1. NONLINEARITY, VOLATILITY AND FRACTIONAL INTEGRATION IN DAILY OIL PRICES: SMOOTH TRANSITION AUTOREGRESSIVE ST-FI(AP)GARCH MODELS

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Abstract

The study aims to extend the GARCH type volatility models to their nonlinear TAR (Tong, 1990) and STAR-based (Terasvirta, 1994) counter parts where both the conditional mean and the conditional variance processes follow TAR and STAR nonlinearity. The paper further investigates the models under their fractional integration and asymmetric power variants. The STAR-based models are LSTAR-LST-GARCH, LSTAR-LST-FIGARCH, LSTAR-LST-FIPGARCH and LSTAR-LST-FIAPGARCH models, which may be easily applied to model and forecast various financial time series. In the empirical section, an application is provided to model the daily returns in WTI crude oil prices considering the regime shifts the crude oil prices were subject to during history. Models are evaluated in terms of their out-of-sample forecasting capabilities with equal forecast accuracy tests and also in terms of various error criteria. The results suggest that volatility clustering, asymmetry and nonlinearity characteristics are modeled more efficiently as compared to their single regime variants, such as the GARCH, FIGARCH and FIAPGARCH models. Further, the out-of-sample results suggest that the LSTAR-LST-FIAPGARCH model provides the best forecasting accuracy in terms of RMSE and MSE error criteria.

Keywords: volatility, oil prices, LSTAR-LST-GARCH, LSTAR-LST-FIGARCH and LSTAR-LST-FIAPGARCH models

JEL Classification: G12, C32, C52, C53

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1. Introduction

The volatility of oil prices is discussed and evaluated in many studies. The determination of volatility in oil prices gains importance for the periods when the oil prices show rapid and unexpected changes. On the other hand, future price forecasts gains relevance especially for the periods of high volatilility. There is a significant literature aiming at improving the capabilities of econometric models to model oil prices.

One point to be taken into consideration is the fact that regime changes that may result from economic factors decrease drastically the forecast capabilities of the models evaluated in the literature. Furthermore, economic factors such that those that led to the cyclical effects that occurred during the 1st and 2nd oil shocks in 1974 and 1979 have significant impacts on the models focusing on the evaluation, modeling and forecasting of oil prices. Consequently, factors such as oil shocks created regime shifts that make the traditional volatility models that do not take regime switching characteristics into consideration obsolete in modeling volatility in oil prices. Accordingly, a significant amount of studies focused on the nonlinear behavior by using different nonlinear techniques. A part of the studies investigating the path followed by the oil prices focus on the GARCH models. With the GARCH models, the effects on both the conditional mean and conditional variance could be tested. In terms of investigating the impacts of intervention on price volatility through derivation of statistical properties to achieve gains in modeling, the FIGARCH models provide significant analytical superiority. FIGARCH modeling provides, in addition to the aspects of intervention on oil prices, an important tool to analyze the finite persistence in oil prices.

The contribution of this paper is to merge regime switching with the GARCH model. LSTAR-LST-GARCH models will be extended to two different versions to introduce fractional integration and asymmetric power properties. These models are LSTAR-LST-PGARCH LSTAR-LST-APGARCH, LSTAR-LST-FIGARCH, LSTAR-LST-FIPGARCH and LSTAR-LST-FIAPGARCH. In the frame of these models, the paper models the oil prices by improving both the conditional mean and the conditional variance with nonlinear time series to achieve improved forecasting and modeling capabilities. The reason behind the usage of LSTAR structure is that the business cycles in the economies reveal different dynamics under different regimes, so that a traditional GARCH model becomes insufficient once the volatility encountered is aimed to be modeled and forecasted for policy purposes. In addition is expected that by augmenting the GARCH models with LSTAR models it would likely bring increase in the forecasting capabilities.

The remainder of the paper is organized as follows. Literature review is given in Part II. Econometric methodology is given in Part III, where both the newly proposed LSTAR-LST-GARCH family models and their fractionally integrated and fractionally integrated asymmetric power versions, LSTAR-LST-FIGARCH and LSTAR-LST-FIAPGARCH models are introduced. Emprical application to oil prices is given in Part IV, while Part V concludes.

2. Literature Review

Among the studies focusing on modeling nonlinarity in oil prices, Raymond and Rich (1997), Clements and Krolzig (2002) and Holmes and Wang (2003) evaluated the oil prices by discussing the impacts of oil shocks on the U.S. and the U.K. in the perspective of business cycles by using the Markov Switching (MS) model. Huang *et al*. (2005) utilized a threshold model to investigate the impacts of oil price changes and their volatility on economic activity.

Adrangi *et al*. (2001) tested the presence of low-dimensional chaotic structure in crude oil, heating oil, and unleaded gasoline futures prices, with their sample starting by the early 1980s. According to the results, chaotic structure is highly persistent and, therefore, it would have strong implications for regulators and short-term trading strategies.

Alvarez and Rodriguez (2008) found that the random walk type behavior in energy futures prices are still an unresolved matter of research; whereas, Alvarez-Ramirez *et al*. (2002) showed that autocorrelation in oil prices diminished over time.

Röthig and Chiarella (2007) used smooth transition regression models to explore nonlinearities in the response of speculators' trading activity to price changes using weekly data sets of the live cattle, corn, and lean hog futures. The authors rejected linearity in all of these markets. Noise traders are commonly divided into two categories: some investors subject to cognitive biases may follow heuristic rules of thumb, exhibit overconfidence and be subject to representativeness bias (Barberis *et al*., 1998; Daniel *et al*., 1998; Hong and Stein, 1999; Gervais and Odean, 2002), whereas other investors may exhibit the disposition effect, which relates to the tendency of investors to sell shares whose price is increasing, while keeping assets that have dropped in value (Oehler *et al*., 2003). McMillan and Speight (2006) found that noise traders typically engage in momentum trading and tend to this behavior when the underlying market is rising, and fundamental traders or arbitrageurs are characterized by heterogeneity, such that movement between different behavior regimes is slow.

3. Econometric Methodology

The ARCH specification of conditional volatility derived by Engle (1982) and further extended to Generalized ARCH (GARCH) model in Bollersev (1986) has found many significant applications in light of modeling the distributional aspects, such as volatility clustering, heavy tails, non-normal distribution in financial markets. Zakoian (1991) proposed the Threshold GARCH (TGARCH) model that aims to capture asymmetric effects of negative and positive shocks with the intuition of capturing different aspects below and above a certain threshold. In terms of smooth transition regressions, several models are developed, which include the commonly applied ST-GARCH model (Franses and van Dijk (2000). Hagerud (1997), Gonzalez-Rivera (1998), Lundbergh and Teräsvirta (1998), Anderson *et al*. (1999), Dufrénot *et al*. (2002) developed and applied the ST-GARCH model. Anè and Rangau (2006) combined the PGARCH model of Ding *et al*. (1993), an extension of the GARCH family models, with the RS-GARCH model and thus developed the RS-APGARCH model. Tse and Tsui (1997) determined the APGARCH model. Brooks *et al*. (2000) showed the leverage effect and the usefulness of including a free power term. Lundbergh and Terasvirta (1998) developed the STAR-STGARCH model that allows nonlinearity in both conditional mean and the conditional variance. Chan and McAleer (2002, 2003) have determined statistical properties in the context of estimation of the STAR-STGARCH family models. Busetti and Manera (2003) used STAR-GARCH models to examine the market interactions in the Pacific Basin Region. Ostermark *et al*. (2004) used STAR type models for modeling Finnish Banking and Finance branch index.

3.1. The LSTAR Models

The STAR models (Luukkonen *et al*., 1988) have very interesting features that allow smooth transition nonlinearity between two or more regimes, where the transition is defined with a continuous, twice-differentiable function defined as the exponential and logistic function following Terasvirta (1994), which further developed the specification, estimation and evaluation methodology based on the LM type tests. The STAR model nests several nonlinear models including the SETAR and TAR models modeled with identity functions. Following Tong (1990), a threshold autoregressive process in the conditional mean is stated as,

$$y_t = \boldsymbol{\alpha}'\tilde{\mathbf{x}}_\mathbf{t} + \boldsymbol{\beta}'\tilde{\mathbf{x}}_\mathbf{t} I\left(s_t\right) + \varepsilon_t \tag{1}$$

where: $\mathbf{x}_t' = \left(y_{t-1}, y_{t-2}, ..., y_{t-p}\right)'$ and $\tilde{\mathbf{x}}_\mathbf{t} = \left[1, \mathbf{x}_t'\right]' = \left(1, y_{t-1}, y_{t-2}, ..., y_{t-p}\right)'$ are the input vectors and the parameter vector I stated as $\phi_i' = \left(\phi_{i,1}, \phi_{i,2}, ..., \phi_{i,p}\right)'$ with *i*=1,2 in a two regime TAR model and the $\varepsilon_t \sim iid\left(0, \sigma_\varepsilon^2\right)$.

The $I\left(s_t\right)$ identity function is a discontinuous function,

$$I(s_t) = \begin{cases} 0 & if \;\; s_t \leq c \\ 1 & if \;\; s_t > c \end{cases} \tag{2}$$

Chan and Tong (1986) extended the TAR model to STAR model by allowing a continuous nonlinear function,

$$y_t = \boldsymbol{\alpha}'\tilde{\mathbf{x}}_\mathbf{t} + \boldsymbol{\beta}'\tilde{\mathbf{x}}_\mathbf{t} F_i\left(s_t\,; \gamma, c\right) + \varepsilon_t \tag{3}$$

Following Luukkonnen *et al*. (1988), Terasvirta (1994) and Granger and Teräsvirta (1993), the STAR models are generalized to allow logistic and exponential functions. The Smooth Transition Autoregressive (STAR) model further developed by Luukkonen *et al.* (1988), Granger and Teräsvirta (1993) and Teräsvirta (1994) aim at nonlinear modeling of the conditional mean by introducing smooth transition between regimes of autoregressive processes based on logistic and exponential functions belonging to squashing functions of neural network models (Bildirici and Ersin, 2013). In the STAR methodology (Teräsvirta, 1994), by taking logistic and exponential functions as transition functions, the LSTAR and ESTAR models are obtained. For a two regime model, the logistic transition function is defined as

$$F\left(s_t;\gamma,c\right)=\left(1+\exp\left\{-\gamma\left(s_t-c\right)\right\}\right)^{-1} \tag{4}$$

where: the input and parameter vectors are defined as in Equation (1).

In the STAR models (Teräsvirta, 1994), the transition variable s_t is selected by LM nonlinearity tests by selecting the optimum lag of the dependent variable, y_{t-d}, which maximizes the explanatory power by choosing the lag that captures the nonlinearity most effectively. Therefore, Equation (5) could be replaced by

$$F\ \left(y_{t-d};\gamma,c\right)=\left(1+\exp\left[-\gamma\left(y_{t-d}-c\right)\right]\right)^{-1} \tag{5}$$

to obtain the following Logistic STAR (LSTAR) representation:

$$y_t=\boldsymbol{\alpha}'\tilde{\mathbf{x}}_{\mathbf{t}}\left(1-F_L\left(y_{t-d};\gamma,c\right)\right)+\boldsymbol{\beta}'\tilde{\mathbf{x}}_{\mathbf{t}}\left(F_L\left(y_{t-d};\gamma,c\right)\right)+\varepsilon_t\,. \tag{6}$$

where: $\varepsilon_t \sim i.i.d.(0,\delta^2)$ is a white noise process with normal distribution. A univariate representation of the LSTAR(p) model with two regimes is

$$\begin{aligned} y_t=&\left(\phi_{1,0}+\phi_{1,1}y_{t-1},\phi_{1,2}y_{t-2}+,...,+\phi_{1,p}y_{t-p}\right)\left(1-\left(1+\exp\left[-\gamma\left(y_{t-d}-c\right)\right]\right)^{-1}\right)+\\ &\left(\phi_{2,0}+\phi_{2,1}y_{t-1},\phi_{2,2}y_{t-2}+,...,+\phi_{2,p}y_{t-p}\right)\left(1+\exp\left[-\gamma\left(y_{t-d}-c\right)\right]\right)^{-1}+\varepsilon_t \end{aligned} \tag{7}$$

where: y_{t-d} is the transition function, *c* is the threshold and γ is the transition parameter that defines the rate of transition which is strictly positive, $\gamma>0$.

For low values of γ, the process has a smooth transition, whereas as $\gamma\rightarrow\infty$ the LSTAR model reduces to the TAR(p) process given in Equation (1). The logistic function F is restricted to values within the interval [0,1]. Further, for $y_{t-d}=c$, the function F =0.5 and as for $y_{t-d}>c$ and $y_{t-d}\ \rightarrow\infty$, the logistic function reaches F =1. Hence, the the LSTAR representation of the dependent variable moves from its first regime representation, $y_t=\phi_1'\tilde{\mathbf{x}}_{\mathbf{t}}+\varepsilon_t$ towards the second regime, $y_t=\left(\phi_1'+\phi_2'\right)\tilde{\mathbf{x}}_{\mathbf{t}}+\varepsilon_t$. Similarly, for $y_{t-d}<c$ and for small values of the transition variable, such that $y_{t-d}\ \rightarrow-\infty$, the process moves smoothly from regime 2 to regime 1. Lastly, for $\gamma=\infty$, the logistic function approaches the identity function, $F=I\left(y_{t-d}\right)$ and the two regime LSTAR model reduces to the Self-exciting threshold autoregressive model, (SE)TAR, (Dick Van Dijk *et al.*, 2002). The models to be analyzed in the study are restricted to the logistic type transition functions and the exponential STAR; the ESTAR representation is not evaluated. Further, the ESTAR model does not nest the TAR representation of the processes. The exponentional

function, $F_E\left(y_{t-d};\gamma,c\right)=\left(1-\exp\left[-\gamma\left(y_{t-d}-c\right)^2\right]\right)$ could be replaced with Equation (5) and substituted into Equation (9) to obtain the ESTAR(p) process as follows:

$$y_t=\left(\phi_{1,0}+\phi_{1,1}y_{t-1},\phi_{1,2}y_{t-2}+,...,+\phi_{1,p}y_{t-p}\right)\left(1-\left(1-\exp\left[-\gamma\left(y_{t-d}-c\right)^2\right]\right)\right)+$$
$$\left(\phi_{2,0}+\phi_{2,1}y_{t-1},\phi_{2,2}y_{t-2}+,...,+\phi_{2,p}y_{t-p}\right)\left(1-\exp\left[-\gamma\left(y_{t-d}-c\right)^2\right]\right)+\varepsilon_t \quad (8)$$

The exponential function $F_E\left(y_{t-d};\gamma,c\right)$ is symmetric around the threshold *c* and for values $y_{t-d}\to\pm\infty$, $F_E\left(y_{t-d};\gamma,c\right)\to 1$ approaches unity in both directions. Therefore, the ESTAR model possesses a middle regime and two outer regimes that are symmetric.

The TAR, LSTAR and ESTAR models given in Equations (1), (8) and (9) allow nonlinearity in the conditional mean processes. Following Hegerud (1997), Gonzales-Rivera (1998) and Lee and Degennaro (2000) the ST-GARCH models allow nonlinear architectures in the conditional variance of a time series. The study aims at introducing fractional integration, symmetric power term and, additionally, asymmetric power terms to obtain LSTAR type nonlinearity in the single regime GARCH models, namely, GARCH, PGARCH, FIGARCH, FIPGARCH, APGARCH and FIAPGARCH volatility models to introduce LSTAR type nonlinearity both in the conditional mean and conditional variance processes. The obtained models to be analyzed in the study are LSTAR-LST-GARCH, LSTAR-LST-PGARCH, LSTAR-LST-APGARCH, LSTAR-LST-FIGARCH, LSTAR-LST-FIPGARCH, LSTAR-LST-APGARCH and LSTAR-LST-FIAPGARCH models, respectively. Therefore, the paper aims at evaluating the nonlinear GARCH models of the ST-GARCH form to introduce their power term, asymmetric power term and fractionally integraged augmentations. The models are evaluated for a long span of data of crude oil prices in the 4th section.

3.2. The LST-GARCH Models

The LST-GARCH Model

The GJR-GARCH model, developed by Glosten *et al*. (1993), is based on the modeling of conditional variance with varying responses to negative and positive lagged innovations with respect to an indicator function. The GJR-GARCH model is represented as

$$\sigma_t^2=w+\alpha\varepsilon_t^2+I\left(\varepsilon_{t-1}\right)\gamma\varepsilon_t^2+\beta\sigma_{t-1}^2 \quad (9)$$

where: $I\left(\varepsilon_{t-1}\right)$ is an indicator function that is $I\left(\varepsilon_{t-1}\right)$=0 if $\varepsilon_{t-1}\geq 0$ and $I\left(\varepsilon_{t-1}\right)$=1 otherwise.

The asymmetry introduced with the γ and the indicator function *I*(.) is called as "the leverage effect"; hence, γ is typically estimated to be positive so that the volatility is increasing proportionately more after negative shocks as compared to the impact of the positive shocks. The identity function will be augmented with the logistic function and the GJR structure will provide a basis for the ST-GARCH models.

Hagerud (1997) and Gonzalez-Rivera (1998) proposed the ST-GARCH model that allows smooth transition between the α and Υ , coefficients of lagged squared error terms of the GJR-GARCH model. A convenient way to formulate the GJR is

$$\sigma_t^2 = w + (1 - I\left|\varepsilon_{t-1} > 0\right|)\alpha_1\varepsilon_{t-1}^2 + (I[\varepsilon_{t-1} > 0])\Upsilon_t\varepsilon_{t-1}^2 + \beta_t\sigma_{t-1}^2 \tag{10}$$

If the *I*(.) indicator function is replaced with the *F*(.) logistic transition function, the Logistic Smooth Transition GARCH (LSTGARCH(1,1)) model is obtained as

$$\sigma_t^2 = w + \left(1 - F\left(\varepsilon_{t-1}\right)\right)\alpha_1\varepsilon_{t-1}^2 + F\left(\varepsilon_{t-1}\right)\Upsilon_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2 \tag{11}$$

where: the transformation function *F* is defined as

$$F(\varepsilon_{t-1}) = \frac{1}{1 + e^{\left(-\theta\varepsilon_{t-1}\right)}}. \tag{12}$$

The logistic function is bounded within the interval [0, 1] and the transition between the regimes occurs from negative to positive values, $\theta > 0$ has non-negativity constraint and the logistic transition function *F* is a monotonic and increasing function of ε_{t-1}. As ε_{t-1} increases from negative values to positive values the impact of $\varepsilon^2{}_{t-1}$ moves proportionately from α_1 to Υ_1. If θ is positive and large enough, the LSTGARCH model transforms into the GJRGARCH model.

By replacing the logistic transformation function with the exponential function, Hagerud (1997) proposed the Exponential Smooth Transformation GARCH (ESTGARCH) model. ESTGARCH(1,1), is differentiated from the LST-GARCH model with the exponential function

$$F\left(\varepsilon_{t-1}\right) = \left(1 - e^{\left(-\theta\varepsilon_{t-1}^2\right)}\right) \tag{13}$$

As a result of formulating the model with the exponential function given in Equation (13), the dynamics of the conditional variance is modeled depending on the size of shocks. This type of nonlinear GARCH formulation is symmetric in terms of the sign of the shocks. The most significant reason for using the exponential function instead of the logistic function is that it allows $F(\varepsilon_{t-1})$ to vary between the boundaries of [0, 1] as $\varepsilon^2{}_{t-1}$ varies between the extreme values.

It is noted that in the ST-GARCH models presented above following the models of Hegerud (1997), Gonzales-Rivera (1998) and Lee and Degennaro (2000), the smooth transition is introduced in the ARCH parameters. Following Anderson *et al*. (1999) and Lundbergh and Terasvirta (2002), the ST-GARCH model may be modeled by allowing the intercept, ARCH and GARCH terms to follow smooth transition between regimes as

$$\sigma_t^2 = \left(1 - F\left(\varepsilon_{t-1}, \theta\right)\right) + \left(w + \beta^* \sigma_{t-1}^2 + \alpha^* \varepsilon_{t-1}^2\right) F\left(\varepsilon_{t-1}, \theta\right) \tag{14}$$

where the parameters of the second regime are denoted by an asterisk.

The conditional volatility may depend on both the size and sign of the shocks on ε_{t-1}. Relative effects of negative and positive shocks of equal magnitudes depend on the amplitude of the conditional volatility of shocks, so that a negative shock may produce a larger shock as compared to the one that a positive shock of similar size could have produced. Negative surprises with large amplitudes may show leverage effects and may lead to volatility with comparatively larger size as compared to the positive surprises (Taylor J.W., 2004).

The LST-FIGARCH Model

The ARCH and GARCH models, developed by Engle (1982) and Bollerslev (1986) respectively, are short memory processes resulting from the fact that the response of a shock to the conditional variance decreases at an exponential rate. On the other hand, the conditional volatility of financial market returns may change slowly over time as a result of long memory characteristics of the financial series. Consequently, the autocorrelation functions may decay at a hyperbolic rate[3].

The Fractionally Integrated GARCH (FIGARCH(1, *d,* 1)) model was developed under these findings by Bollersev and Mikkelsen(1996) and Baillie *et al*. (1996) as an extension of the GARCH model to account for long memory. In this section, we first evaluate fractional integration in a GARCH setting to evaluate long memory in conditional variance. Afterwards, smooth transition type nonlinearity setting will be introduced to the evaluated FIGARCH and FIAPGARCH models.

Assume that a time series following a random walk process in its conditional mean and its conditional variance, σ_t^2 =Var$\left(\varepsilon_t \middle| \Omega_{t-1}\right)$, where the information set up to time *t*-1 is denoted by Ω_{t-1}, follows a FIGARCH(1,d,1) process

$$\left(1-\beta L\right)\sigma_t^2 = \alpha + \left(\left(1-\beta L\right) - \left(1-\phi L\right)\left(1-L\right)^d\right)\left(\left|\varepsilon_{t-1}\right| - \gamma\varepsilon_{t-1}\right)^2 \tag{15}$$

or alternatively,

$$\sigma_t^2 = \alpha + \beta\sigma_{t-1}^2 + \left(\left(1-\beta L\right) - \left(1-\phi L\right)\left(1-L\right)^d\right)\left(\left|\varepsilon_{t-1}\right| - \gamma\varepsilon_{t-1}\right)^2 \tag{16}$$

where: z_t is assumed to be normally distributed *N*(*0*,*1*) white noise process,

[3] Bollerslev and Mikkelsen (1996) develop the necessary conditions for FIGARCH model and note that for a well defined FIGARCH model, all the coefficients in the infinite ARCH representation must be non-negative (see: Bollerslev and Mikkelsen, 1996, p. 159). FIGARCH models are further discussed in Nelson and Cao (1992) and Conrad and Haag (2006), following these studies, nonnegativity constraints on parameters of FIGARCH processes are relaxed and shown that for p=2 the second lag of conditional variance can become negative. Futher, Conrad and Haag (2006) allow conditions so that even if all parameters are negative (apart from d), the conditional variance can be nonnegative for FIGARCH models following the inequality constraints of Conrad and Haag (2006).

$$z_t \sim N(0,1) = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{z_t^2}{2}\right) \tag{17}$$

The FIGARCH(1, *d,* 1) model nests the GARCH model if *d* = 0 and the IGARCH model of Engle and Bollerslev (1986) if *d* = 1, the estimated fractional integration parameter. The fractional integration parameter *d* is 0<d<1 and as $d \to 0$ $(d \to 1)$ the model has short memory (long memory) characteristics. For alternative specifications of the FIGARCH model, readers are referred to Karanasos *et al*. (2004), Giraitis *et al*. (2004) and Zaffaroni (2004).

The ST-FIGARCH model, which generalizes the ST-GARCH type nonlinearity to account for fractional integration, is represented as follows

$$\begin{aligned}\sigma_t^2 &= \omega + \left(1 - F\left(\varepsilon_{t-s}, \gamma\right)\right)\alpha\sigma_{t-1}^2 + \beta F\left(\varepsilon_{t-s}, \gamma\right)\sigma_{t-1}^2 \\ &+ \left[\left[1 - \alpha L\left(1 - F\left(\varepsilon_{t-s}, \gamma\right)\right) - \beta L F\left(\varepsilon_{t-s}, \gamma\right)\right] - (1 - \phi L)(1 - \phi L)^d\right] u_t^2\end{aligned} \tag{18}$$

for $\gamma \neq 0$ the width of the volatility clusters and α and β characterizes the dynamics of the conditional volatility. The range of the cluster of volatility changes between $F(.) = 0$ and $F(.) = 1$. The constant term takes on values between $\varphi = \omega/(1-\alpha)$ and $\varphi = \omega/(1-\beta)$ based upon whether the conditional volatility is regime dictated by $F(.) = 0$ and $F(.) = 1$.

Accordingly, since in the ST-GARCH model the constant term ranges between the extreme regimes, the level of conditional volatility changes in different regimes (Kılıç, 2010). If the transition function $F(.)$ is the logistic function of the following form

$$F(\varepsilon_{t-1}) = \frac{1}{1 + e^{\left(-\theta\varepsilon_{t-1}\right)}} \tag{19}$$

the model becomes the logistic smooth transition FIGARCH (LST-FIGARCH) model.

The LST-FIAPGARCH Model

Tse (1998) introduced the FIAPGARCH model, which combines long memory property of Baillie, Bollerslev, and Mikkelsen (1996) FIGARCH model with Asymmetric Power GARCH (APGARCH) model of Ding et al. (1993) by extending the FIGARCH model to account for different asymmetric dynamics. Accordingly, the fractionally integrated APGARCH model is represented as

$$(1 - \beta L)\sigma_n^\delta = \omega + \left((1 - \beta L) - (1 - \phi L)(1 - L)^d\right)\left(|\varepsilon_{n-1}| - \gamma\varepsilon_{n-1}\right)^\delta \tag{20}$$

where: L denotes the lag operator, *d* is the $0 \le d \le 1$ functional differencing parameter, β denotes the autoregressive parameters, ϕ represents the moving average parameters of the conditional variance equation, δ represents the optimal power transformation, γ represents the asymmetry parameter and $|\gamma|$< 1 ensures that positive and negative innovations of the same size can have asymmetric effects on the conditional variance (Conrad, Rittler and Rotfuss; 2010).

Further, after imposing the restrictions δ =2 and d=0, the FIAPGARCH model reduces to the AGARCH model; whereas, if the restriction δ =2 is applied, the model reduces to FIAGARCH, and if d=0 the model reduces to APGARCH model.

The ST-ARCH modeling methodology developed by Hegerud (1997), Gonzales-Rivera (1998), Lee and Degennaro (2000) allows smooth transition type nonlinearity in ARCH parameters and the ST-GARCH models of Anderson *et al*. (1999) and Lundbergh and Terasvirta (2002) accept a modeling structure that in addition to the ARCH terms the intercept and the GARCH terms are extended to be modeled with smooth transition type nonlinearity in different regimes.

Accordingly, following the ST-FIGARCH model structure, the smooth transition fractionally integrated asymmetric power GARCH model denoted by ST-FIAPGARCH is obtained by allowing the smooth transition type nonlinearity between two FIAPGARCH models in two different regimes defined as

$$\sigma_t^{\delta} = \omega + \left(1 - F\left(\varepsilon_{t-s}, \gamma\right)\right)\alpha\sigma_{t-1}^{\delta} + \beta F\left(\varepsilon_{t-s}, \gamma\right)\sigma_{t-1}^{\delta}$$

$$+\left[\left[1 - \alpha L\left(1 - F\left(\varepsilon_{t-s}, \gamma\right)\right) - \beta L F\left(\varepsilon_{t-s}, \gamma\right)\right] - (1 - \phi L)(1 - \phi L)^{d}\right]\left(|\varepsilon_{n-1}| - \gamma\varepsilon_{n-1}\right)^{\delta} \tag{21}$$

If the transition function $F(.)$ is defined as a logistic function bounded between 0 and 1,

$$F(\varepsilon_{t-1}) = \frac{1}{1 + \mathbf{exp}\left(-\gamma\varepsilon_{t-1}\right)} \tag{22}$$

the obtained model becomes the LST-FIAPGARCH model.

3.3. The LSTAR-GARCH Models

The STAR-GARCH models, evaluated by Lundberg and Terasvirta (1999, 2000) and Franses Neele and van Dijk (1998) and further examined by Chan and McAleer (2001) are time series models with STAR type nonlinear processes in the conditional mean with heteroskedasticy given as GARCH errors. Consider the following STAR model (Terasvirta, 1994) with two regimes:

$$y_t = \left(\phi_1 + \sum_{i=1}^{r} \phi_{1i} y_{t-i}\right)\left(1 - F\left(s_t; \gamma, c\right)\right)\left(\phi_2 + \sum_{i=1}^{r} \phi_{2i} y_{t-i}\right) F\left(s_t; \gamma, c\right) + \varepsilon_t \tag{23}$$

where

$$F\left(s_t; \gamma, c\right) = \frac{1}{1 + e^{-\gamma(s_t - c)}} \tag{24}$$

defined with the logistic function. By allowing GARCH errors

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-1}^2 + \sum_{i=1}^{r} \beta_i \sigma_{t-1}^2 \quad (25)$$

the model is called Logistic Smooth Transition Autoregressive GARCH (LSTAR-GARCH) model. As the information matrix of the log-likelihood function of STAR-GARCH is block diagonal, the parameters in the conditional mean and conditional variance equations can be estimated separately, as in the case of ARMA-GARCH. The general GARCH properties are expected to hold (Chan and McAleer, 1999).

The LSTAR-LST-GARCH Model

The Smooth Transition Autoregressive (STAR) model further developed by Luukkonen *et al.* (1988), Granger and Terasvirta (1993) and Terasvirta (1994) aims at nonlinear modeling of the conditional mean by introducing smooth transition between regimes of autoregressive processes based on logistic and exponential functions belonging to squashing functions. The link between the LSTAR-LST-GARCH and their neural network augmentations are discussed in Bildirici and Ersin (2013). In the STAR models, commonly applied transition functions are logistic and exponential functions and the relevant models are called LSTAR and ESTAR models. The LSTAR–LSTGARCH model is a model that allows STAR type nonlinearity in both the conditional mean and the conditional variance and is developed on the basis of the following STAR model. The error terms follow smooth transition in the GARCH process

$$\sigma_t^2 = \left(w_1 + \sum_{i=1}^{p} \alpha_{1i} \varepsilon_{t-1}^2 + \sum_{i=1}^{r} \beta_{1i} \sigma_{t-i}^2 \right) \left(1 - F\left(s_t; \varsigma, c\right)\right) + \left(w_2 + \sum_{i=1}^{p} \alpha_{2i} \varepsilon_{t-1}^2 + \sum_{i=1}^{r} \beta_{2i} \sigma_{t-i}^2 \right) F\left(s_t; \varsigma, c\right) \quad (26)$$

with the transition function

$$F\left(s_t; \varsigma, c\right) = \frac{1}{1 + e^{-\varsigma(s_t - c)}} \quad (27)$$

where: ς is the parameter defining the speed of transition and *c* is the threshold coefficient. The model will be extended to LSTAR-LST-FIGARCH model.

The LSTAR-LST-FIGARCH Model

Assume that a time series following a random walk process in its conditional mean and its conditional variance, $h_t = \mathrm{var}\left(\varepsilon_t \middle| \Omega_{t-1}\right)$, where the information set up to time *t*-1 is denoted as Ω_{t-1}, follows a FIGARCH(1,d,1) process

$$\left(1 - \beta L\right)\sigma_t^2 = \omega + \alpha\left(\left(1 - \beta L\right) - \left(1 - \phi L\right)\left(1 - L\right)^d\right)\left(\left|\varepsilon_{t-1}\right| - \gamma \varepsilon_{t-1}\right)^2 \quad (28)$$

or alternatively

$$\sigma_t^2 = \omega + \beta h_t + \alpha\left(\left(1 - \beta L\right) - \left(1 - \phi L\right)\left(1 - L\right)^d\right)\left(\left|\varepsilon_{t-1}\right| - \gamma \varepsilon_{t-1}\right)^2 \quad (29)$$

where: z_t is assumed to be normally distributed *N*(*0,1*) white noise process

$$z_t \sim N(0,1) = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{z_t^2}{2}\right) \tag{30}$$

The LSTAR-LST-FIGARCH model, on the other hand, allows the conditional mean process to follow a LSTAR representation and assumes that long memory in two different regimes is also governed by a logistic smooth transition type process as given in Equation (27). The model is represented as

$$\begin{aligned}\sigma_t^2 &= \left(\omega_{0,1} + \beta_1 h_{t,1} + \alpha_1\left((1-\beta_1 L) - (1-\phi_1 L)(1-L)^{d,1}\right)\left(|\varepsilon_{t-1}| - \gamma_1 \varepsilon_{t-1}\right)^2\right) \times \left(1 - F(s_t;\varsigma,c)\right) \\ &\left(\omega_{0,2} + \beta_2 h_{t,2} + \alpha_2\left((1-\beta_2 L) - (1-\phi_2 L)(1-L)^{d,2}\right)\left(|\varepsilon_{t-1}| - \gamma_2 \varepsilon_{t-1}\right)^2\right) \times \left(F(s_t;\varsigma,c)\right)\end{aligned} \tag{31}$$

where: $\gamma \neq 0$ is non-zero and defines the width of the volatility clusters and similarly, α and β characterizes the dynamics of the conditional volatility.

The range of the cluster of volatility changes between $F(s_t;\varsigma,c) = 0$ and $F(s_t;\varsigma,c) = 1$. The constant term takes on values between $\varphi = \omega/(1-\alpha)$ and $\varphi = \omega/(1-\beta)$ based upon whether the conditional volatility in each regime is dictated by the transition function as $F(s_t;\varsigma,c) \to 0$ and/or $F(s_t;\varsigma,c) \to 1$. Accordingly, since in the ST-GARCH model the constant term ranges between the extreme regimes, the level of conditional volatility changes in different regimes (Kılıç, 2010). If the transition function $F(s_t;\varsigma,c)$ is logistic function

$$F(s_t;\varsigma,c) = \frac{1}{1+e^{-\varsigma(s_t - c)}} \tag{32}$$

the model becomes the logistic smooth transition FIGARCH (LST-FIGARCH) model.

The LSTAR-LST-FIAPGARCH Model

The FIAPGARCH model introduced by Tse (1998) combines the long memory property of Baillie, Bollerslev, and Mikkelsen (1996) FIGARCH model with the Asymmetric Power GARCH (APGARCH) model of Ding *et al.* (1993) by extending the FIGARCH model to account for different asymmetric dynamics. Accordingly, the fractionally integrated APGARCH model is represented as

$$(1-\beta L)\sigma_t^\delta = w + \alpha\left((1-\beta L) - (1-\phi L)(1-L)^d\right)\left(|\varepsilon_{t-1}| - \gamma \varepsilon_{t-1}\right)^\delta \tag{33}$$

where the conditional mean follows a random walk process, L denotes the lag operator, *d* is the $0 \leq d \leq 1$ functional differencing parameter, β denotes the autoregressive parameters, ϕ represents the moving average parameters of the conditional variance equation, δ represents the optimal power transformation, γ represents the asymmetry parameter and $|\gamma|$< 1 ensures that positive and negative innovations of the same size can have asymmetric effects on the conditional variance (Conrad et al. 2010).

Further, after imposing the restrictions δ =2 and d=0, the FIAPGARCH model reduces to AGARCH model; whereas, if the restriction δ =2 is applied, the model reduces to FIAGARCH, and if d=0 the model reduces to APGARCH model. Moreover, the APGARCH model also nests the Power GARCH model. By restricting the γ parameter to zero, the FIAPGARCH model reduces to the FIPGARCH model as

$$(1-\beta L)\sigma_t^{\delta} = w + \alpha\left((1-\beta L)-(1-\phi L)(1-L)^{d}\right)\left(|\varepsilon_{t-1}|\right)^{\delta} \tag{34}$$

By imposing δ =2 and d=0 restrictions, the FIPGARCH model reduces to AGARCH model and similarly, if the restriction δ =2 is applied, the model reduces to FIAGARCH. If a restriction on the fractional differention parameter is applied as d=0, the model reduces to the power GARCH model. It should be noted that the obtained power GARCH model is also referred to as the Nonlinear GARCH model, or the NGARCH model introduced by Higgins and Bera (1992), where the conditional standard deviation is raised to the power as a function of lagged conditional standard deviations and the lagged absolute innovations to the same power (Bollerslev, 2010). Therefore, a PGARCH(1,1) process could be defined as

$$\sigma_t^{\delta} = w + \alpha\varepsilon_{t-1}^{\delta} + \beta\sigma_{t-1}^{\delta} \tag{35}$$

which reduces to the standard GARCH(1,1) process for δ =2. The FIAPGARCH and FIGARCH models could be easily extended to STAR type nonlinearity. The ST-ARCH modeling methodology developed by Hegerud (1997), Gonzales-Rivera (1998), Lee and Degennaro (2000) allows smooth transition type nonlinearity in ARCH parameters and the ST-GARCH models of Anderson (1999) and Lundbergh and Terasvirta (2002) accept a modeling structure, which allows the intercept and the GARCH terms to be extended with smooth transition type nonlinearity in addition to the ARCH terms.

The LSTAR-LST-FIAPGARCH model is a process with nonlinear STAR type dynamics both in the conditional mean and conditional variance processes. By assuming that the conditional mean process could be modeled with a well-defined LSTAR process, the nonlinearity in the conditional variance is obtained by allowing smooth transition based LST-FIAPGARCH process that allows smooth transition between two FIAPGARCH processes in two different regimes as

$$\begin{aligned}(1-\beta L)\sigma_t^{\delta,i} &= \left(\omega_{0,1}+\alpha_1\left((1-\beta_1 L)-(1-\phi_1 L)(1-L)^{d,1}\right)\left(|\varepsilon_{t-1}|-\gamma_1\varepsilon_{t-1}\right)^{\delta,1}\right)\times\left(1-\left(F(s_t;\varsigma,c)\right)\right)\\ &+\left(\omega_{0,2}+\alpha_2\left((1-\beta_2 L)-(1-\phi_2 L)(1-L)^{d,2}\right)\left(|\varepsilon_{t-1}|-\gamma_2\varepsilon_{t-1}\right)^{\delta,2}\right)\times\left(F(s_t;\varsigma,c)\right)\end{aligned} \tag{36}$$

In the model, the transition function $F(s_t;\varsigma,c)$ is defined as a logistic function bounded between 0 and 1 similar to the model in Equation (33), which governs the transitions between two regimes both in the conditional mean and conditional variance processes

$$F\left(s_t;\varsigma,c\right)=\frac{1}{1+e^{-\varsigma\left(s_t-c\right)}} \tag{37}$$

It should be noted that the LSTAR-LST-FIAPGARCH model nests the models discussed above. By restricting the γ =0 in Equation (36), the LST-FIAPGARCH process reduces to the LST-FIPGARCH process, as

$$\begin{aligned}\left(1-\beta L\right)\sigma_t^{\delta,i}=&\left(\omega_{0,1}+\alpha_1\left(\left(1-\beta_1 L\right)-\left(1-\phi_1 L\right)\left(1-L\right)^{d,1}\right)\left(\left|\varepsilon_{t-1}\right|\right)^{\delta,1}\right)\times\left(1-\left(F\left(s_t;\varsigma,c\right)\right)\right)\\&+\left(\omega_{0,2}+\alpha_2\left(\left(1-\beta_2 L\right)-\left(1-\phi_2 L\right)\left(1-L\right)^{d,2}\right)\left(\left|\varepsilon_{t-1}\right|\right)^{\delta,2}\right)\times\left(F\left(s_t;\varsigma,c\right)\right)\end{aligned} \tag{38}$$

Similarly, by applying certain restrictions, the following models could be easily obtained. By applying the d=0 restriction, the LST-FIAPGARCH model given in Equation (36) reduces to the LST-APGARCH, and the LST-FIPGARCH model given in Equation (38) reduces to the LST-PGARCH. The asymmetric power GARCH based and the symmetric power GARCH based LSTAR-LST-APGARCH, LSTAR-LST-PGARCH, LSTAR-LST-FIAPGARCH and LSTAR-LST-FIPGARCH models have interesting properties, since they allow power terms to be estimated differently for two regimes. In addition to allowing different ARCH and GARCH parameter estimates in two regimes, the proposed models allow different degrees of fractional integration and long memory characteristics. The power terms are allowed to take different values to be estimated for each regime in Equation (38) and different asymmetric power structures in the LSTAR-LST-FIAPGARCH model given in Equation (36). It should be noted that the smooth transition between two regimes in the proposed models are defined by taking the transition parameter s_t as $s_t = y_{t-1}$ to allow the conditional mean process to be modelled simultaneously with the conditional variance processes by allowing the nonlinearity to be governed with the logistic function given in Equation (37).

4. Econometric Results

4.1. Data

The data set evaluated in the study aims at evaluating the above-mentioned nonlinear volatility models to achieve, if possible, improvement in in-sample and out-of-sample forecasting. In order to test the forecasting performances of the above-mentioned models, oil price volatility is calculated by using the daily closing prices of oil price covering the interval January 2nd,1986 - May, 13th, 2014, covering a sample of n=7254 observations. The crude oil prices are the *spot prices for crude oil* downloaded from the database of U.S. Energy Information Administration (EIA) and are availabile at: http://www.eia.gov/dnav/pet/pet_pri_spt_s1_d.htm. Data is first transformed into natural logarithms, and to obtain the daily returns the data is first-differenced as $y=ln(P_t)-ln(P_{t-1})$, where P_t represents the crude oil prices. In the process of estimating the models, the last ten observations are left for out-of-sample forecasts.

4.2. Econometric Results: Model Evaluation

In the first stage, from among the GARCH family models, we selected basic GARCH models (GARCH, PGARCH and APGARCH) and basic Fractionally Integrated GARCH models (FIGARCH, FIPGARCH and FIAPGARCH), taken as baseline models estimated for evaluation purposes. The results are given in Table 1. The included models have different characteristics to be evaluated; namely, fractional integration, asymmetric power and fractionally integrated asymmetric power models, namely, GARCH, APGARCH, FIGARCH and FIAPGARCH models.

It is noticed that all volatility models perform better than the FIGARCH model in light of Log Likelihood criteria. If AIC and SIC criteria are evaluated, the lowest AIC and SIC are calculated as -4.926904 and -4.921206 and are obtained for the FIGARCH model. The sum of ARCH and GARCH parameters is lower than 1 for the APGARCH, FIGARCH and FIAPGARCH model. Further, we noted that the stability condition is not obtained for the PGARCH and APGARCH models, since the sum of ARCH and GARCH parameters is equal to or more than 1.

Table 1

The Single Regime GARCH Models

	Baseline GARCH Models			Baseline Fractionally Integrated GARCH Models		
	GARCH	PGARCH	APGARCH	FIGARCH	FIPGARCH	FIAPGARCH
Cst(M)	0.000571*** (2.75)	0.0000895 (1.62)	0.00049** (2.35)	0.000599*** (2.83)	0.000601*** (2.85)	0.000568*** (2.689)
Cst(V)	0.00000048*** (4.45)	0.0000131*** (3.03)	0.0000089** (1.98)	0.0000467* (1.82)	0.000112 (1.51)	0.0001172*** (1.51)
d-Figarch		-		0.461*** (7.78)	0.452251*** (8.90)	0.445489*** (8.38)
ARCH	0.0642*** (7.61)	0.0759***† (11.85)	0.0743***† (8.64)	0.2716*** (3.49)	0.2751*** (3.68)	0.2929*** (3.50)
GARCH	0.9282*** (106.1)	0.9320*** (162.61)	0.9330*** (116.9)	0.6178*** (5.67)	0.6155*** (6.166)	0.6231*** (5.71)
APACH (Gamma1)		-	0.148973*** (2.649)		-	0.100840* (1.89)
APACH (Delta)		1.2282*** (8.07)	1.2273*** (10.46)		1.8825*** (24.73)	1.8493*** (23.27)
LogL	17890.302	17903.15	17906.287	17875.881	17877.364	17880.239
AIC	-4.931	-4.93	-4.935	-4.926904	-4.926761	-4.927554
SIC	-4.926	-4.92	-4.928	-4.921206	-4.919164	-4.919956

*Note: The significant parameters at 1%, 5% and 10% significance levels are denoted by ***, ** and *, respectively. LogL is the log-likelihood, AIC and SIC are the Akaike and Bayesian information criteria, ARCH(1-2) is the second order ARCH-LM test and SB is the sign bias test statistics. For the ARCH-LM and SB tests, the probabilities are reported in brackets. The t statistics are given in parantheses. † denotes that the stability condition is not satisfied since the sum of ARCH and GARCH parameters are equal to or larger than 1.*

Once the models are introduced with fractional integration, the condition is satisfied for the last three models, the FIGARCH, FIPGARCH and FIAPGARCH models in addition to the baseline GARCH models. For the fractionally integrated models, the differentiation parameters are estimated at 0.461 for the FIGARCH model, 0.452251 for the FIPGARCH model and 0.445489 for the FIAPGARCH model. Further, the power parameters are estimated ats 1.228 for the PGARCH, 1.227383 for APGARCH; whereas, the power terms are comparatively larger and are calculated at 1.882553 and 1.84931 for the FIPGARCH and FIAPGARCH models, respectively. The obtained results are largely affected by the fact that the oil prices are subject to significant increases due to oil shocks and economic crises. Since the power parameter estimates are not larger than 2, we accepted the results and took the baseline models to be evaluated in comparison to the LSTAR-based nonlinear volatility models. Furthermore, oil prices are determined to reflect other decisions, both political and economic. Certain evaluations could be easily derived considering the long memory characteristics that show the persistence of shocks. Considering the results obtained for the models estimated for the whole period, the fractional difference parameter estimations show that the effects of the shocks could be eliminated at a slow rate, although they satisfy the persistence condition that is the d-FIGARCH parameter being equal to or larger than 0.5. Although the results suggested that no strong persistence existed in oil prices, once the volatility is modeled with nonlinear models, the regime specific results migh show different results in addition to regime specific asymmetric power structures in the conditional volatility.

In this section, the results of LSTAR-LST-GARCH models will be evaluated. In the study, the LSTAR-GARCH models are estimated by the BFGS algorithm by assuming that the error terms follow Gaussian distribution. Statistical inference regarding the empirical validity of two-regime switching process was carried out by using nonstandard LR tests (Davies, 1987). The non-standard LR test is statistically significant and this suggests that linearity is strongly rejected. Further, Lukkonnen *et al*. (1988) LM nonlinearity tests are conducted. It is concluded that remaining nonlinearity in the error terms is rejected. During modelling, the first 5 lags of daily change in oil prices are evaluated with Luukkonnen *et al*. (1988) and Terasvirta (1994) F type linearity tests against STAR type nonlinearity. Accordingly, to avoid the nuisance parameter problem as evaluated in Davies (1988), 3rd order Taylor expension method is applied following Terasvirta (1994). For details, readers are referred to Terasvirta (1994)[4].

The estimation results for the LSTAR-LST-GARCH models with no fractional integration are reported in Table 2. The transition parameters are estimated at 4.24, 4.58 and 2.26 for the models with LSTAR-LST-GARCH, LSTAR-LST-PGARCH and LSTAR-LST-APGARCH models, respectively. The stability condition is noted to be larger than 1 for the first regimes of the LSTAR-LST-GARCH and LSTAR-LST-APGARCH model; whereas, the condition holds for both regimes in LSTAR-LST-PGARCH model which also possess a very large ARCH+GARCH value for the first regime.

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[4] *The threshold variable is the one that maximized the F statistic following the Terasvirta (1994) STAR type nonlinearity F test sequence. The threshold variable selected is given in Table 2. The results are not given in the paper to save space. The additional tables are available from the authors upon request.*

Table 2

The LSTAR-LST-GARCH Type Nonlinear Model Estimates

	LSTAR-LST-GARCH		LSTAR-LST-PGARCH		LSTAR-LST-APGARCH	
Cst(M)	-0.003339*** (-7.427)	0.0289*** (53.65)	-0.002650** (-6.48)	0.025083*** (55.42)	-0.006201*** (18.16)	0.11853*** (34.44)
Cst(V)	0.0000021 (1.19)	0.00000117* (1.79)	0.0000036 (0.56)	0.000134 (1.38)	0.000352 (1.133)	0.0002245 (0.895)
d-Figarch	-	-	-	-	-	-
ARCH	0.0802***† (3.656)	0.1186** (2.29)	0.0769*** (3.80)	0.1668***† (3.14)	0.3154***† (6.442)	0.11532** (2.0753)
GARCH	0.9211*** (45.36)	0.8579*** (15.94)	0.9163*** (54.29)	0.8586*** (31.57)	0.7101*** (8.945)	0.84451** (1.9758)
APACH (Gamma1)				-	0.47936* (1.735)	0.4901* (1.8079)
APACH (Delta)			1.578515*** (3.98)	0.958722*** (4.88)	1.1189*** (5.0725)	1.0603*** (3.996)
Transition speed	4.23772** (2.40)		4.58435** (2.01)		2.26435*** (4.00)	
Threshold	0.04769*** (10.85)		-0.05229*** (-10.39)		-0.03904*** (-13.20)	
Threshold variable	P_{t-4}		P_{t-4}		P_{t-2}	
LogL	3801.5		3861.233		3864.69	
AIC	-5.384		-5.390		-5.392	
SIC	-5.369		-5.365		-5.368	
ARCH(1-2)	0.403[0.67]		0.225[0.80]		0.41249 [0.6621]	
SB test	1.24[0.21]		0.176[0.86]		1.16[0.2001]	

*Note: The significant parameters at 1%, 5% and 10% significance levels are denoted by ***, ** and *, respectively. LogL is the log-likelihood, AIC and SIC are the Akaike and Bayesian information criteria, ARCH(1-2) is the second order ARCH-LM test and SB is the sign bias test statistic. For the ARCH-LM and SB tests, the probabilities are reported in brackets. The t statistics are given in parantheses. † denotes that the stability condition is not satisfied since the sum of ARCH and GARCH parameters is equal to or larger than 1.Th threshold variable is selected among the lags 1-5 of the dependent variable (based on the AIC information criterium) that maximized the rejection of linearity as suggested by Luukkonen et al. (1988) and Terasvirta (1994) STAR model selection methodology based on F tests.*

If an overlook is provided without discussing the further results given in Tables 3 and 4, the results show that once the fractional integration structure is applied, and after the incorporation of long memory characteristics, for the FIGARCH, FIPGARCH and FIAPGARCH type of nonlinear GARCH models proposed in the study, the explanatory power of the models are largely increased. In addition, the fractional integration based models in Table 3 satisfy the stability conditions. Further, to model oil prices and the inherited long memory, fractional integration deserves special attention. However, if the researcher requested not to apply fractional integration to LSTAR-GARCH type models, following Bildirici and Ersin (2013), the neural network augmented versions of the LSTAR-LST-GARCH type models without fractional integration provide another method to improve forecasting accuracy. Hence, the results suggest that the oil prices are subject to long memory charateristics and without applying it into modeling, the forecasting power also diminishes drastically.

In Table 2, the threshold parameter estimates are statistically significant and are estimated at 0.04769, -0.05229 and -0.03904 for the three evaluated models. The power parameter estimates for the first and second regimes of the LSTAR-LST-APGARCH model are statistically significant and are estimated at 1.18 and 1.06 for the first and second regimes. This result shows that asymmetry is statistically higher for the first regime as compared to the second regime that holds once the threshold variable passes the threshold coefficient of -0.04. A similar conclusion holds for the symmetric power model, the LSTAR-LST-PGARCH model, for which the power terms are estimated at 1.58 and 0.96. The results point towards a striking feature of the nonlinear models in Table 2, vis-à-vis their linear counterparts, the single regime PGARCH and APGARCH models for which the power parameter estimates are 1.228 and 1.227, respectively. Further, as compared to the baseline GARCH models reported in Table 1, the lower AIC and SIC statistics reported for their nonlinear counterparts suggest that LSTAR-LST-GARCH type nonlinear models in Table 2 provide better in-sample modeling performances, which could be considered as a sign of better goodness-of-fit. For further conclusions, the out-of-sample results will provide important insights in comparing the modeling performances of the analyzed models. If the nonlinear models are compared among themselves, the asymmetric power type LSTAR-LST-APGARCH model provides the best in-sample forecasting results (AIC=-5.392, SIC=-5.368). The sign bias tests suggest that at 5% significance tests, no sign-bias exists in both three nonlinear models. The ARCH-LM tests suggest that no ARCH effect exists in the residuals, suggesting that the models are successful in filtering the ARCH type heteroskedasticity in the crude oil prices.

After introducing fractional integration to the models represented above, the results obtained for the nonlinear LSTAR-LST-FIGARCH type models are reported in Table 3, which are the FIGARCH type augmentations of the models in Table 2. One overlook could reveal the fact that the differentiation parameter estimates in both regimes of the LSTAR-LST-FIGARCH is larger than 0.50, suggesting a sign of long-memory in crude oil prices. A similar result holds only for the first regime of the LSTAR-LST-FIPGARCH, the model obtained after introducing the power term to the LSTAR-LST-FIGARCH model. Further, after introducing the asymmetric power transformation, the fractional integration parameter estimates become lower than 0.50 for both regimes in the model given in column 3. If the results are compared with the results obtained for the baseline FIGARCH, FIPGARCH and FIAPGARCH models for which the differentiation parameters were estimated between 0.44 and 0.46, the results of the nonlinear models represent a different structure, since the dynamics in regimes 1 and 2 which occur below and above the threshold estimates suggest lower estimates for the differentiation parameters[5]. As a result of obtaining different ARCH and GARCH

[5] *After the introduction of the asymmetric power terms, similar to the result with single regime models, the fractional difference parameter estimates are calculated with large positive values. For the 1st and 2st regime of LSTAR-LST-FIGARCH model, fractional difference parameter is calculated at 0.515 and 0.60, suggesting strong persistence whereas, in LSTAR-LSTFIAPGARCH, for the both regimes, the parameters are estimated at 0.198 and 0.374 and*

parameters in two regimes, the stability condition is also affected. The condition is satisfied for both regimes of the LSTAR-LST-FIGARCH model; however, the sum of ARCH and GARCH parameters is closer to 1 than the sum of parameter estimates in regime 2. The stability condition is satisfied for both regimes of LSTAR-LST-FIPGARCH. After the introduction of both fractional integration and asymmetric power terms in two regimes, the stability condition is close to 1 but not larger than 1 in both regimes of the LSTAR-LST-FIAPGARCH model.

Table 3

The LSTAR-LST-FIGARCH Models, Models with Fractional Integration

	LSTAR-LSTFIGARCH		LSTAR-LSTFIPGARCH		*LSTAR-LSTFIAPGARCH*	
Cst(M)	-0.00654*** (16.93)	0.01536*** (42.75)	-0.002371*** (5.261)	0.023815*** (117.4)	-0.003648*** (16.08)	0.029183*** (54.32)
Cst(V)	0.000061 (1.502)	0.000052** (2.322)	0.0000417 (0.6595)	0.001** (2.073)	0.001342*** (1.771)	0.001349 (1.098)
d-Figarch	0.5154*** (9.180)	0.5985*** (4.0123)	0.5001*** (5.612)	0.4150*** (3.487)	0.1981*** (4.088)	0.3704** (2.286)
ARCH	0.1172* (1.708)	0.3412** (1.96)	0.3091*** (3.040)	0.3831*** (2.708)	0.3966*** (7.509)	0.2422*** (3.119)
GARCH	0.8701*** (4.165)	0.6411*** (10.63)	0.6809*** (7.246)	0.6127*** (3.318)	0.6032*** (8.478)	0.7222*** (106.638)
APACH (Gamma1)			-	-	0.603595*** (2.844)	0.3941** (1.986)
APACH (Delta)			1.637593*** (7.14)	1.441863*** (6.22)	1.101373*** (9.053)	1.0879*** (3.973)
Transition speed	11.10264 (1.0546)		43.13430 (0.5813)		4.42996 (1.53)	
Threshold	0.03540*** (12.51)		0.02316*** (19.04)		0.03973*** (6.29)	
Transition variable	P_{t-2}		P_{t-3}		P_{t-2}	
LogL	3860.65		3801.31		3846.58	
AIC	-5.49256		-5.721247		-5.402227	
SIC	-5.47856		-5.691622		-5.380009	
ARCH(1-2)	0.18110 [0.8342]		0.050078 [0.9512]		0.24079 [0.7860]	
SB test	1.55024		0.05920		0.04968	

*Note: The significant parameters at 1%, 5% and 10% significance levels are denoted by ***, ** and *, respectively. LogL is the log-likelihood, AIC and SIC are the Akaike and Bayesian information criteria, ARCH(1-2) is the second order ARCH-LM test and SB is the sign bias test statistic. For the ARCH-LM and SB tests, the probabilities are reported in brackets. The t statistics are given in parantheses. † All fractionally integrated nonlinear models satisfied the stability condition. ˠ The threshold variable is selected among the lags 1-5 of the dependent variable (based on the AIC information criterium) that maximized the rejection of linearity as suggested by Luukkonen et al. (1988) and Terasvirta (1994) STAR model selection methodology.*

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are less than 0.50. Further, for the LSTAR-LSTFIPGARCH model, in the first regime the parameter is estimated at 0.500, but at 0.415 in regime 2. For the models that do not have fractional integration structure, stability condition is satisfied for the 1st regimes only.

Compared to the baseline GARCH models and LSTAR-LST-GARCH type models reported in Table 1 and 2, the lowest AIC and SIC statistics are reported for the nonlinear models in Table 3, which also take fractional integration into consideration. The lowest AIC and SIC statistics were reported for the LSTAR-LST-APGARCH model suggesting the best in-sample forecasting results (AIC=-5.392, SIC=-5.368). After the introduction of fractional integration, the AIC and SIC statistics becomes significantly lower for the LSTAR-LST-FIAPGARCH model (AIC=-5.402227, SIC=-5.380009), followed by the LSTAR-LST-FIGARCH model (AIC=-5.49256, SIC=-5.47856) compared to the baseline GARCH models in addition to the improved performance of the LSTAR-LST-GARCH models. The lowest AIC and SIC values are reported for the LSTAR-LST-FIPGARCH model (AIC=-5.49256, SIC=-5.47856). According to the results, the best in-sample modeling performances are achieved for the nonlinear models with fractional integration. The sign bias tests suggest that no sign-bias exists in all the models in Table 3. Further, the ARCH-LM tests suggest that the fractionally integrated models are successful in capturing the ARCH type heteroskedasticity in the modeled crude oil price data. It should be noted that, in addition to the in-sample performances of the models, the out-of-sample results will provide important information in terms of generalization capabilities of the analyzed models[6].

Compared to the results obtained in Tables 1 and 2, after the incorporation of long memory characteristics, the models given in Table 3 provided the stability condition in terms of the sum of ARCH and GARCH parameters. As discussed before, the explanatory power of the models is largely following the fractional integration specification. Further, the explanatory power of the models with fractional integration provided a drastic increase in terms of more negative AIC and SIC statistics. The results are under the influence of the fact that the oil prices are better modeled once long memory and fractional integration is taken into consideration. It should be noted that out-of-sample performances will show additional measures in terms of modeling oil prices and forecasting accuracy.

In terms of evaluating the results with an economic policy perspective, the LSTAR-LST-FIGARCH model shows that the impacts of shocks are likely to last longer since there is strong level of persistence, especially for the 2nd regime. Since large positive values are estimated for both regimes, the results suggest a certain amount of persistence in both regimes. For the LSTAR-LST-FIPGARCH model, the persistence effect is comparatively lower only in the second regime; whereas, for the LSTAR-LST-FIAPGARCH model that showed the lowest performance among the fractional integration model group in Table 3 persistence effect is quite low. With a political perspective, by considering the best two models analyzed, these findings show that

[6] *Further, linearity is tested with Luukkonnen* et al. *(1988) tests against the STAR type nonlinearity in the residuals. The results suggest no remaining nonlinearity. Accordingly, LSTAR-GARCH and LSTAR-LST-GARCH family models provided significant achievements in terms of modeling oil prices as compared to the basic GARCH family models given in Table 1. The results are not reported to save space; however, they may be obtained from the authors upon request.*

the policy makers should keep the interventions at modest levels to avoid large fluctuations in oil prices, an important commodity also in production, considering the high persistence in oil prices. The long memory results also coincide with these results that show the temporary effects of these shocks and their dependence on relative levels of oil prices to the threshold values. Therefore, since the impacts of the oil shocks could be permanent, the interventions on oil prices should be kept at restrained levels.

One point that cannot be overlooked is that the out-of-sample forecasting capabilities provide significant findings in terms of generalization and modeling capabilities of the models analyzed. The models are evaluated for their generalization capabilities in the out-of-sample with RMSE error criteria for 2, 5 and 10 days ahead. The results are given in Table 4, which constitutes of 6 nonlinear models to be compared with the baseline single regime models. First group is the GARCH family models; namely, the GARCH, PGARCH, APGARCH, FIGARCH, FIPGARCH and FIAPGARCH models, respectively. The second group is the nonlinear counterparts of the models, namely, the LSTAR-LST-GARCH family that with both LSTAR type nonlinearity in the conditional mean and in the conditional variance and lastly, the LSTAR-LST-FIGARCH family with STAR type nonlinearity and fractional integration characteristics. The number of models to be analyzed totals to 12 models, respectively.

Table 4

Out-of-Sample Forecasting Results of the Models

GARCH Family of Models (2 Days Ahead)						
	GARCH	*PGARCH*	*APGARCH*	*FIGARCH*	*FIPGARCH*	*FIAPGARCH*
RMSE	0.01105	0.01131	0.01097	0.01133	0.01135	0.01113
LSTAR-LST-GARCH Family of Models (2 Days Ahead)						
	LSTAR-LST-GARCH	*LSTAR-LST-PGARCH*	*LSTAR-LST-APGARCH*	*LSTAR-LST-FIGARCH*	*LSTAR-LST-FIPGARCH*	*LSTAR-LST-FIAPGARCH*
RMSE	0.003339	0.00265	0.00258	0.002689	0.001795	0.001794
Relative % change:	-69.78	-76.57	-76.48	-76.27	-84.19	-83.88
GARCH Family of Models (5 Days Ahead)						
	GARCH	*PGARCH*	*APGARCH*	*FIGARCH*	*FIPGARCH*	*FIAPGARCH*
RMSE	0.007381	0.007542	0.007334	0.007554	0.007564	0.007428
LSTAR-LST-GARCH Family Models (5 Days Ahead)						
	LSTAR-LST-GARCH	*LSTAR-LST-PGARCH*	*LSTAR-LST-APGARCH*	*LSTAR-LST-FIGARCH*	*LSTAR-LST-FIPGARCH*	*LSTAR-LST-FIAPGARCH*
RMSE	0.01007	0.01048	0.01041	0.01043	0.004673	0.004673
Relative % change:	36.43	38.96	41.94	38.07	-38.22	-37.09
GARCH Family Models (10 Days Ahead)						
	GARCH	*PGARCH*	*APGARCH*	*FIGARCH*	*FIPGARCH*	*FIAPGARCH*
RMSE	0.007756	0.007756	0.007758	0.007757	0.007757	0.007755

LSTAR-LST-GARCH Family Models (10 Days Ahead)						
	LSTAR-LST-GARCH	*LSTAR-LST-PGARCH*	*LSTAR-LST-APGARCH*	*LSTAR-LST-FIGARCH*	*LSTAR-LST-FIPGARCH*	*LSTAR-LST-FIAPGARCH*
RMSE	0.01237	0.0126	0.0112	0.012	0.004201	0.004201
Relative % change:	59.49	62.45	44.37	54.70	-45.84	-45.83

Note: RMSE is the root mean squared error. The relative percentage change shows the percentage decrease (if negative) in RMSE as compared to its baseline counterpart model.

The first group consists of the baseline models; the GARCH, PGARCH, APGARCH, FIGARCH, FIPGARCH and FIAPGARCH models, respectively. In the first group, for 2 days ahead, the lowest RMSE is achieved for the APGARCH model (RMSE=0.01097); followed by the GARCH and FIAPGARCH models (RMSE= 0.01105 and 0.01113, respectively). The second group corresponds to the LSTAR-LST-GARCH family models; the nonlinear counterparts of the first group. For 2 days ahead, the LSTAR-LST-GARCH family showed significant improvement over the baseline GARCH family. The lowest RMSE is achieved by the LSTAR-LST-FIAPGARCH model with RMSE=0.001794, and is closely followed by the LSTAR-LST-FIPGARCH model with RMSE=0.001795. The obtained RMSE statistics corresponded to a -83.88% decrease in terms of RMSE and to a -84.19% decrease in RMSE for the LSTAR-LST-FIAPGARCH and LSTAR-LST-FIPGARCH models, as compared to their baseline FIAPGARCH and FIPGARCH counterparts. Further, the RMSE statistics for the LSTAR-LST-GARCH, LSTAR-LST-PGARCH, LSTAR-LST-APGARCH and LSTAR-LST-FIPGARCH models were calculated at 0.003339, 0.00265, 0.00258 and 0.002689, which represented a relative percentage change of -69.78, -76.57, -76.48 and -76.27over their single regime baseline models.

The results showed that the lowest RMSE values were calculated for the LSTAR-LST-FIAPGARCH and LSTAR-LST- FIPGARCH for two days ahead forecasts. For 5 days ahead, the lowest RMSE statistics were obtained again by the LSTAR-LST-FIPGARCH and LSTAR-LST-FIAPGARCH models (RMSE=0.004673 for both models), which showed -38.22 and -37.09% relative change (decrease) in RMSE over their baseline single regime versions. On the other hand, the RMSE values for the LSTAR-LST-GARCH, LSTAR-LST- PGARCH and LSTAR-LST-APGARCH were calculated at 0.01007, 0.01048, 0.01041 and 0.01041; which are relatively by 40% on the average larger than the baseline single regime versions. The results show that although for short periods (2 days ahead) the LSTAR-LST-GARCH modeling showed drastic improvement over the baseline models for forecasting oil prices, for a longer span (5 days ahead) the modeling of these models with fractional integration and power terms benefited them in terms of forecasting. Ten days ahead analysis corresponded to the largest span of forecasting practice. According to the results, the nonlinear models with fractional integration and power terms, namely, the LSTAR-LST-FIPGARCH and LSTAR-LST-FIAPGARCH provided the best out-of-sample performances (RMSE=0.004201 for both models), which represented a relative percentage decrease by -45.84 and -45.83 over their baseline single regime counterparts. The results coincided with those obtained with the 5 days ahead analysis, where the single regime models of GARCH, PGARCH and APGARCH showed improvement over their nonlinear counterparts. Further, the nonlinear models with fractional integration and power terms provided significant improvement over the single regime and LSTAR type models without fractional integration.

5. Policy Implications and Evaluation of the Results Obtained by the Nonlinear Analyzed Models

The oil price in macroeconomic perspective, which is an important variable for explaining business cycles and economic growth, exhibits a large volatility. Increases in oil prices affects the industrial production, business cycles, current account deficits and financial markets through various channels. Although the volatility of oil price could certainly affect the values of financial oil-based derivatives, it should be emphasized that volatility of oil price should not have generally any significant impact on the values of most real options and the related investment decisions. Oil prices may not always adjust instantaneously to new information; on the other hand, low liquidity and infrequent trading in imperfect markets could cause a delay in response to new information (see for similar suggestion; McMillan and Speight, 2006; Monoyios and Sarno, 2002; Lee, Liu and Chiu; 2008). The results obtained by the models suggest that the increases in volatility are generally short-lived but its effect is relatively long-lived. The results under influence of large positive values that are estimated for both regimes suggest persistence in both the regimes. On the other hand, according to our results, the positive fractional coefficients determined that the impacts of shocks are persistent. The government and/or policy makers should keep the interventions at modest levels to avoid large fluctuations and persistence in oil prices. Although the long memory results emphasized the temporary effects of these shocks, the impacts of the oil shocks could be permanent, so that the interventions to oil prices by the government should be limited.

According to the results obtained in this study, the oil prices possess important characteristics, such as nonlinearity, asymmetry, threshold effects and persistence effects that should lead the policy makers and the researchers to evaluate the policies to be applied with great care; hence, the nonlinear volatility models that incorporate fractional integration and power terms capture the data generating process more effectively, therefore providing important tools for policies. Firstly, with a political perspective, since nonlinearity, asymmetry and long memory characteristics play crucial role in oil prices, the policies aimed at stabilizing the volatility of this crucial commodity may have destabilizing effects on the production and on financial markets through various channels. This result translates itself to different derivatives and the economy, and this destabilization effect is largely under the influence of persistence in oil prices and also in the external shocks that oil prices are subject to. Thus, policies might have destabilizing effects if persistence is not taken into consideration. Secondly, the estimation sample in the study corresponded to a period with large oil shocks and economic crises periods; whereas, the out-of-sample subsample (the last 10 observations) consists of a stabilized period, since it is a general approach lo leave the last observations for out-of-sample analyses in the literature. However, though significant improvement in terms of forecasting is achieved for the nonlinear models, the performance of these models would improve drastically once the forecasts would have been obtained for a period of unexpected changes in oil prices, since the nonlinear models benefit the policy makers especially once large fluctuations of oil occurred, by passing the estimated thresholds of these models. Therefore, the results should be taken as follows. The nonlinear models provided in the study would improve the forecasting capabilities or the toolbox of the policy maker and the researchers, however, the results should always be evauated with care considering the fluctuations caused by the drastic changes in oil prices and, also, by keeping in mind that oil prices are subject to regime changes and threshold effects that lead to different dynamics.

6. Conclusion

The study aimed at investigating oil prices by focusing nonlinearity and asymmetry in addition to fractional integration that causes interesting characteristics in oil prices. Considering the nonlinear data generating process in addition to regime-specific volatility, the study focused on introducing fractionally integrated models in addition to models with no fractional integration. Further, the STAR-GARCH and ST-GARCH models are extended to LSTAR-LST-GARCH and LSTAR-LST-FIGARCH processes which also include their power term and asymmetric power term augmentations; namely, the LSTAR-LST-FIPGARCH and LSTAR-LST-FIAPGARCH models. The models constituted a family of 6 nonlinear models which are evaluated vis-à-vis their baseline GARCH counterparts that are single regime models in nature. The models suggested in the study showed that the impacts of shocks possess significant persistence and nonlinearity characteristics in oil prices. Therefore, in terms of economic policy perspective, policy makers should avoid interventions on oil prices and the interventions should be kept limited, if possible, to avoid the persistent characteristics of shocks in oil prices. However, the nonlinear models provided in the study should be considered as important tools for the policy makers, under the condition that the threshold effects and nonlinearity that lead to regime changes should be evaluated with great care.

The conclusions in the empirical section are derived as follows. The models with fractional integration and power terms provided significant gains as compared to simple GARCH models. Once the LSTAR nonlinearity is introduced in the conditional mean and conditional volatility processes of oil price series, the LSTAR-LST-GARCH family of models augmented the forecasting capabilities, especially after considering the long memory and persistence characteristics by introducing fractional integration to the LSTAR nonlinear architecture of the analyzed models. Further, the LSTAR-LST-GARCH models failed to satisfy the stability condition for certain regimes. The results showed that after applying fractional integration structures to the proposed nonlinear models, the incorporation of long memory characteristics, the explanatory powers of the models are largely increased in addition to achieving the stability condition for all the models with fractional integration. Accordingly, oil prices inherited strong persistence, and fractional integration deserves special attention. It should be noted that if the researchers requested not to apply fractional integration to LSTAR-LST-GARCH models to model oil prices, following Bildirici and Ersin (2013), the neural network augmented versions of the LSTAR-LST-GARCH models without fractional integration provide another methodology to achieve improvement in terms of forecasting accuracy.

According to the results, to avoid large fluctuations and destabilizing policies, the policies that aim at oil markets should take nonlinearity and asymmetry into consideration, in addition to long memory and persistence characteristics of oil prices by also considering the external shocks that oil prices had been subject to in the history, and possibly, in the future.

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