1.

THE MARKOV-SWITCHING GRANGER CAUSALITY OF ASIA-PACIFIC EXCHANGE RATES

Jing-Tung WU[1]

Abstract

This paper examines the Granger causality relationships of the Asia-Pacific exchange rates. We employ Markov-switching vector autoregressive model to capture the dynamic linkages between them. Empirical examination processes use the nominal and real exchange rates of Chinese Yuan, Japanese yen, New Taiwan dollar and South Korean Won.

The results of Markov-switching Granger causalities differ remarkably from the conventional linear model and provide more accurate measurement. We find the Markov-switching Granger causalities between exchange rates, and they vary with respect to the sample lag period. The mutual relationships between Asia-Pacific exchange rates are profound and lasting. This is suggested that we should use the nonlinear model to recheck the Granger causalities of exchange rates, and capture the fluctuations of Asia-Pacific exchange rates.

Keywords: Asia-Pacific, exchange rate, Granger causality, Markov-switching vector autoregressive model

JEL Classification: F31; F37; C34

I. Introduction

In the foreign exchange market, the value of a currency is expressed in its relation to other currencies; foreign exchange rates are mutually determinate. A number of fundamental factors affect the value of currency; these usually involve the country's macroeconomic prospects, inflation, money supply, trade balances, as well as each country's central bank policy.

Causal information can always be shown to be useful, and can produce better forecasts. The causal link between foreign exchange rate changes has been extensively documented by earlier studies, but some inconclusive evidence still remains. Granger causality has primarily been studied within the linear vector autoregressive (VAR)

[1] *Department of Finance, Ming Chuan University. E-mail: jtwu@mail.mcu.edu.tw.*

model. However, some researchers have noted that the results from Granger causality tests tend to be sensitive with respect to the sample period. One approach in dealing with this problem is to separate the whole period under consideration into several periods. By the effort of Thoma (1994), a rolling-window technique was used to investigate the stability of the result, but in the absence of priori information about the breaking points of causal relationships, this approach has not worked well at all (Christopoulos & $Leo'n$ -Ledesma, 2008).

Recently, we have seen an increasing interest in studying financial time series as nonlinear models, in contrast to linear models. This is due to a number of studies having found significant nonlinear behavior in financial markets. A variety of non-linear models have been considered as alternatives to the conventional linear models. For example, Hamilton & Susmel (1994) provided evidence that the Markov-switching autoregressive conditional heteroscedasticity model (Markov-switching ARCH) of exchange rates outperforms the ARCH and generalized ARCH (GARCH) models. Engle & Hamilton (1990), LeBaron (1992), Bekaert & Gray (1996) and Engle & Hakkio (1996) all documented regime shifts in major foreign exchange rates.

Krolzig (1997, 2000) extended the Markov-switching to VAR cases, and developed an approach to predict multiple time series subjects to Markovian shifts in the regime. This methodology treats the changes in causality as random events governed by the Markov process, such that it could capture the instability of Granger causality between variables. It is a variant of a VAR model where the intercept, parameter coefficients and error term are all subject to Markov-switching. This paper adopts Krolzig's methodology to investigate the Granger causality of Asia-Pacific exchange rates and to enhance the application of the model. By this paper, we reveal the profound and lasting relationships between Asia-Pacific exchange rates.

The remainder of this paper is structured in the following manner: section 2 presents the Markov-switching VAR model to capture the Granger causality of Asia-Pacific exchange rates. Section 3 describes the empirical analysis. Finally, section 4 presents the conclusions of this paper.

II. The Model

II.1.Granger causality

Granger causality is the most widely used concept of causality in time series econometrics. It was introduced by Granger (1969) and has primarily been studied within linear VAR models. For example, consider a matrix consisting of two variables ($Y_t = [y_{1,t}, y_{2,t}]$) that follows an autoregressive system:

$$Y_t = \alpha + \sum_{i=1}^{l} \beta_i Y_{t-1} + \varepsilon_t \tag{1}$$

where: l is the lag period. The Granger approach as to whether $y_{1,t}$ causes $y_{2,t}$ involves seeing how much of the current $y_{1,t}$ can be explained by past values of $y_{1,t}$, and then seeing whether adding lagged values of $y_{2,t}$ offers significant prediction power of current values of $y_{1,t}$. Now bi-directional causality is frequently the case, i.e. $y_{2,t}$ Granger causes $y_{1,t}$ and $y_{1,t}$ also Granger causes $y_{2,t}$. The Granger tests for causality involve the estimation of the following equations:

$$y_{1,t} = \alpha_1 + \sum_{i=1}^{l_{11}} \gamma_{1i} y_{1,t-i} + \sum_{i=1}^{l_{12}} \eta_{1i} y_{2,t-i} + \varepsilon_{1,t} \tag{2}$$

$$y_{2,t} = \alpha_2 + \sum_{i=1}^{l_{21}} \gamma_{2i} y_{1,t-i} + \sum_{i=1}^{l_{22}} \eta_{2i} y_{2,t-i} + \varepsilon_{2,t} \tag{3}$$

Now we state that $y_{1,t}$ and $y_{2,t}$ are the foreign exchange rates for the Chinese Yuan (CNY), Japanese Yen (JPY), New Taiwan Dollar (TWD) and South Korean Won (KRW), respectively. Linear causal relationships can be inferred from Eqs. (2) and (3). If $\sum \eta_{1i} = 0$, then Eq. (2) implies that past $y_{2,t}$ has no influence on $y_{1,t}$, that is $y_{2,t}$ does not Granger cause $y_{1,t}$. If $\sum \gamma_{2i} = 0$, then Eq. (3) implies that past $y_{1,t}$ has no influence on $y_{2,t}$. On the other hand, if $\eta_{1i} \neq 0$ for some values of i, then it may be implied that $y_{2,t}$ Granger causes $y_{1,t}$. If $\gamma_{2i} \neq 0$ for some values of i, then it may be implied that $y_{1,t}$ Granger causes $y_{2,t}$. We could use this test to explore the mutual relationship between two exchange rates or even among several exchange rates.

Other methods have also been used to study Granger causality, including rolling-window technique (Thoma, 1994), autoregressive moving average (Boudjellaba et al., 1994), logistic smooth transition vector autoregressive (Christopoulos & $Leo'n$-Ledesma, 2008), and GARCH models ($W\alpha'niak$, 2012).

II.2. Markov-switching VAR model

Markov-switching model is designed to capture discrete changes in the economics that generates the financial time series. It is a popular time-varying volatility model which allows researchers to identify separate joint normal distributions for the time-series returns. It was popularized by Hamilton's (1989) study of business cycle dynamics. In recent years, researchers and professionals have extensively used Markov-switching model to model the regime-switching nature of economic processes, predict future price movements and to capture volatility dynamics. It might result in forecast devices superior to time-invariant linear models.

The regime generating process is assumed as a Markov chain with finite regimes s_t. Empirical studies have documented that exchange rate returns tend to be leptokurtic; thus, a Student t-distribution is suggested for the innovations. Hamilton (1994) found that the leptokurtic can result from the mixture of normal distribution. In a two regimes case, if $s_t = 1$, then the process is in regime one, while $s_t = 2$ means that the process is in regime two. It is always assumed that s_t cannot be observed directly; its operation is only inferred through the observed behavior of y_t; the parameters necessary to fully describe the probability law governing y_t are the two regime transition probabilities, p_{11} and p_{22}.

The transition of regimes is stochastic; one is never sure, whether or not there will be a change of the regime. The probability transition matrix P controls the probability of a switch from regime one to regime two, or from regime two to regime one. The probability transition matrix is usually assumed to remain constant. The Markov-switching model assumes that the probability of a change in the regime depends on the past only through the value of the most-recent regime. The probability transition matrix P is defined as follows:

$$P = \begin{pmatrix} \Pr(s_t = 1 \mid s_{t-1} = 1) = p_{11}, \Pr(s_t = 1 \mid s_{t-1} = 2) = p_{21} \\ \Pr(s_t = 2 \mid s_{t-1} = 1) = p_{12}, \Pr(s_t = 2 \mid s_{t-1} = 2) = p_{22} \end{pmatrix} \tag{4}$$

The data y_t are summarized by six population parameters:

$$\theta = (\mu_1, \mu_{2,} \sigma_{1,} \sigma_2, p_{11}, p_{22}) \tag{5}$$

The estimation procedure uses the Gaussian maximum likelihood method, and the optimal model is selected by Bayesian or minimum Akaike information criteria (AIC). If the Markov chain is presumed to be ergodic, one can use the unconditional probabilities. Engle and Hamilton (1990) investigated the change in exchange rates and found that Markov-switching model is a good approximation to the underlying processes, with excellent predictive power of nonlinearities in time series; see Hamilton (1994) for further details. It can yield some improvements compared to the traditional linear model. So the debates arise on choosing the optimal value of state in modeling the dynamics of exchange rates. According to the previous discussions of it, we could not find a standard distribution theory applicable for evaluating the model.

Krolzig (1997, 2000) extended the Markov-switching to VAR cases, and developed an approach to predict multiple time series subjects to Markovian shifts in the regime. He assumed that the changes of parameters are stochastic and governed by an unobserved Markov chain; we then change Eq. (1) to (6) and represent it by Markov-switching VAR model:

$$Y_t = \alpha_{s_t} + \sum_{i=1}^{l} \beta_{i_{s_t}} Y_{t-1} + \varepsilon_{s_t} \tag{6}$$

with $\varepsilon_{s_t} \sim N(0, \sum s_t)$, variance matrix $\sum$ is finite and non-negative. Krolzig (1997, 2000) argued that an attractive feature of Markov-switching VAR models is that multi-step forecasts can be obtained when the autoregressive parameters are regime-invariant. This feature allowed us to analyze the implications of the predictability and Granger causality of regimes on the optimal prediction. If the autoregressive parameters are regime invariant, the optimal predictor is linear in the information set. In fact, due to the findings of previous studies, the non-linear behavior of foreign exchange rates, the optimal predictor generally lacks the property of being a linear predictor. This paper estimates a Markov-switching VAR model in order to identify the dynamic impact of interrelated time series of exchange rates. In a two exchange rates' case, the model is given as follows:

$$y_{1,t} = \alpha_{1s_t} + \sum_{i=1}^{l_{11}} \gamma_{1i_{s_t}} y_{1,t-i} + \sum_{i=1}^{l_{12}} \eta_{1i_{s_t}} y_{2,t-i} + \varepsilon_{1_{s_t}} \tag{7}$$

$$y_{2,t} = \alpha_{2s_t} + \sum_{i=1}^{l_{11}} \gamma_{2i_{s_t}} y_{1,t-i} + \sum_{i=1}^{l_{12}} \eta_{2i_{s_t}} y_{2,t-i} + \varepsilon_{2_{s_t}} \tag{8}$$

Eqs. (7) and (8) could easily extend to three or more exchange rates cases. The exploration of Markov-switching Granger causality has also attracted consideration from other researchers; for example, Otranto (2005) illustrated a multi-chain Markov-switching model to describe the Granger causality between variables. Psaradakis *et al*. (2005) proposed a model of Markov-switching Granger causality in order to identify the dynamic impact of interrelated time series. Droumaguet *et al*. (2015) used the Markov-switching VAR model to analyze a system of monthly US data on money and income. This paper adopts this idea and applies it to the analysis of foreign exchange rates. In order to derive a more precise prediction, the approach is based on the Markov-switching VAR model.

Besides the Markov-switching model, other well-known nonlinear models are applied to related studies, such as the threshold autoregressive model (TAR) proposed by Tong (1978) and Tong & Lim (1980), while Teräsvirta (1994) presented the idea of a smooth transition autoregressive model (STAR). There is no clear consensus regarding the forecasting abilities of these models. However, the estimation of regime change should be inferred by the unknown threshold variables or unknown lagged variables when we apply the TAR model and STAR model. Unlike these two models, the Markov-switching model uses the probabilities' transition matrix to control the probability of a switch to a different regime. It is not necessary to rely on unknown variables; thus, the selection of those explanation variables would not be a problem for the Markov-switching model. The benefit makes it easier to apply to empirical studies.

Nonlinearly in foreign exchange rates has long been widely recognized in the literature. In this section, we outline the econometric procedure employed in order to model regime shifts in the dynamic relationship between the exchange rates.

III. Empirical Results

III.1. Preliminary analysis of the data

The monthly data are taken from the *Taiwan Economic Journal* (TEJ), with the sample period covering May 1996 through December 2014, yielding a total of 222 observations. Foreign exchange rate volatilities are logarithmically transformed with the percentage change compared to the last month.

Four major Asia-Pacific exchange rates are employed: CNY, JPY, KRW and TWD. The exchange rates are all against U.S. Dollar. We select these four exchange rates based on the heavy trade between countries and higher correlation between the exchange rates. This paper carries out the Augmented Dickery-Fuller (ADF) and Phillips-Perron (PP) unit root tests to verify whether the series is stationary. The results show that returns of foreign exchange rates are stationary at the 1% significance level. Since these tests are well-known and have been widely used in researches, the mathematical details are not presented here. Table 1 reports the base statistics and unit root tests in the return series.

Table 1

The statistics of logarithmic monthly foreign exchange rate return

	CNY	JPY	KRW	TWD
Mean	-0.121	0.046	0.147	0.060
Max	2.683	8.522	37.110	8.171
Min	-2.085	-16.305	-16.570	-5.750
S.D.	0.451	3.137	4.577	1.620
ADF	-5.424***	-15.031***	-14.490***	-13.035***
PP	-12.477***	-15.032***	-14.490***	-13.021***

This paper investigates the Granger causality of the exchange rates. By using the method proposed by Brock *et al*. (1987) to test time series independence (BDS test) we evaluate all the possible relations between exchange rates and their lagged returns. The BDS test can be applied to check whether the time series residuals are independent and identically distributed (*iid*). Table 2 shows the testing results of the exchange rates; it rejects the *iid* hypothesis. They are appropriate for further statistical analysis.

Table 2

The BDS statistics

Dimension	CNY	JPY	KRW	TWD
2	0.051***	0.007	0.066***	0.020***
3	0.116***	0.017**	0.111***	0.032***
4	0.160***	0.020**	0.132***	0.041***
5	0.191***	0.020**	0.140***	0.046***
6	0.212***	0.015	0.138***	0.048***

7	0.226***	0.013	0.129***	0.047***
8	0.239***	0.012	0.116***	0.043***

Note: We first choose a distance value of 0.7 for testing shorter dimensions, then increase the distance value and get the similar results. Here only report the results of distance value 0.7.

III.2. Estimates of Granger causality

We examine the Granger causality of Asia-Pacific exchange rates. In order to investigate the influence of interest rate, we provide another time series. The original one is the returns of exchange rates (nominal exchange rate); the other one is the returns of exchange rates after being adjusted by interest rate (real exchange rate). To investigate the changes in causality over the sample period, this paper considers different lagged periods. Table 3 shows the results of one, two and three months lag for the linear Granger causality test. It is found that the Granger causality of TWD and KRW is significant at almost all horizons. At each lagged period, regarding the TWD, Granger causes the KRW. Conversely, with KRW, Granger causes the TWD at all times except at one month lagged.

In the case of TWD and CNY, TWD and JPY, all situations are non-significant. TWD does not Granger causes CNY and JPY at each lagged period and moves in the opposite direction.

Table 3

Granger causality p-values of different lagged periods

Granger causality	1 month		2 months		3 months	
	nominal	real	nominal	real	nominal	real
TWD→CNY	0.3886	0.4114	0.4145	0.1870	0.4957	0.7992
CNY→TWD	0.7452	0.5475	0.9274	0.5916	0.9663	0.2709
TWD→JPY	0.7015	0.4207	0.9612	0.7407	0.6252	0.5106
JPY→TWD	0.8025	0.7612	0.7872	0.8857	0.5923	0.4703
TWD→KRW	0.0000	0.0559	0.0000	0.0046	0.0000	0.0033
KRW→TWD	0.2309	0.0001	0.0101	0.0002	0.0066	0.0002

The results of TWD and KRW cases show that the Granger causalities are fairly persistent. We may conclude that TWD, and KRW has bi-directions Granger causality at both nominal and real exchange rates, while the other exchange rates do not. This finding is not consistent with numerous studies that Granger causality tests tend to be sensitive with respect to changes in the sample periods (Psaradakis *et al.*, 2005; Christopoulos & $Leo'n$ -Ledesma, 2008). This phenomenon may be due to the returns behavior of exchange rates are different with the other sample.

III.3. Estimates of the Markov-switching VAR model

We use the Markov-switching VAR model to investigate the potential changes in causality at different situations. Parameters are estimated by the maximum likelihood (ML) method; the likelihood function is evaluated by an iterative filtering algorithm. Tables 4, 5 and 6 contain the results of TWD←→CNY, TWD←→JPY and TWD←→KRW models, respectively. Each table present estimated results from one month lag to three months lag. By using Eqs. (7) and (8), we could select the optimal lag length with the criterion of minimum log likelihood.

Table 4 illustrates the estimated results of the Markov-switching VAR model for TWD←→CNY. The log likelihood value of lag for one month without interest rate adjusted is well below those for two or three months. The probabilities of remaining in the current regimes (p_{11} and p_{22}) are 0.93 and 0.57; both are significant. This means the expected durations of the regimes are 14.53 and 2.35 months, respectively. The results imply that by using the Markov-switching VAR model to separate the sample into two regimes, the coefficients of the parameters differ in both regimes and bring sizable deviations between them. We carefully compare this result with the linear Granger causality, if we use the original method to measure, it may cause some error. Here is the benefit that Markov-switching model brings about in figuring out the relation between foreign exchange rate and fundamental factors.

At lags of one month to three months TWD does not see Granger causing CNY at all. Conversely, CNY Granger causes the TWD at one month and three months lag; some coefficients are significant. This differs from the linear cases, which lack Granger causality at both directions.

Table 5 lists the estimated results of the Markov-switching VAR model for TWD←→JPY. The log likelihood value of lag for three months without interest rate adjusted is below those for one and two months. The probabilities of remaining in the current regimes (p_{11} and p_{22}) are 0.92 and 0.74; p_{11} is significant. This means the expected durations of the regimes are 12.23 and 3.78 months, respectively.

TWD Granger causes JPY at two and three months lag. Conversely, JPY Granger causes TWD at one and three months lag. The bi-direction Granger causality appears at three months lag, which is the optimal lagged length we selected based on the criterion of minimum log likelihood. This differs from the linear cases, which lack Granger causality at both directions.

Table 6 shows the estimate results of the Markov-switching VAR model for TWD←→KRW. The log likelihood value of lag for two months without interest rate adjusted is below those for one and three months. The probabilities of remaining in the current regimes (p_{11} and p_{22}) are 0.92 and 0.71; both are significant. This means the expected durations of the regimes are 12.03 and 3.40 months, respectively. The large estimated transition probability of the regime one implies that the regime is highly persistent.

TWD Granger causes KRW at one, two and three month lags. Conversely, with KRW, Granger causes TWD at one, two and three month lags. The bi-direction Granger causality appears at the lag for each month. This is only slightly different from the linear case.

From Tables 4 to 6, the optimal lagged length in TWD←→CNY case is one month; for the TWD←→JPY case, it is three months; for the TWD←→KRW case, it is two months. They are all nominal exchange rates series.

Figure 1 displays the regime one smoothed probabilities of nominal TWD←→CNY Markov-switching Granger causality lags for 3 months. Figure 2 is TWD←→JPY case; Figure 3 is TWD←→KRW case. The regime one durations of nominal TWD←→CNY is

10.15 months, TWD←→JPY is 12.23 months, TWD←→KRW is 10.04 months. However, the regime two durations of nominal TWD←→CNY is 8.84 months, TWD←→JPY is 3.78 months, TWD←→KRW is 9.66 months. The durations of the regime one are similar, but it emerges a wide variety of regime two. Figures 1~3 displays the significance of two regimes Markov-switching Granger causalities, and the variations of duration. They provide the identical information with table 4 to 6.

Table 4

TWD←→CNY Results from Markov switching VAR model

Parameters/lags			1 month				2 months				3 months	
	$regime_n$	$regime_n$	$regime_r$	$regime_r$	$regime_n$	$regime_n$	$regime_r$	$regime_r$	$regime_n$	$regime_n$	$regime_r$	$regime_r$
α_1	*-0.05 (0.09)*	*-0.75 (0.43)*	-0.02 (0.36)	0.01 (0.94)	0.02 (0.10)	0.55 (0.44)	0.04 (0.20)	-0.01 (0.17)	0.06 (0.14)	0.06 (0.21)	-0.10 (0.17)	0.10 (0.21)
γ_{11}	*0.21*** (0.07)*	*-0.33 (inf)*	0.33*** (0.11)	-0.16 (0.21)	-0.04 (0.07)	0.28 (0.19)	0.39*** (0.09)	-0.30* (0.16)	0.43*** (0.10)	-0.17 (0.15)	0.44*** (0.10)	-0.30** (0.12)
γ_{12}					-0.25*** (0.07)	0.09 (0.24)	-0.01 (0.14)	0.06 (0.18)	-0.29 (0.08)	0.22 (0.17)	-0.09 (0.08)	0.01 (0.25)
γ_{13}									0.28*** (0.07)	-0.18 (0.18)	0.16*** (0.05)	-0.15 (0.08)
η_{11}	*0.42 (0.27)*	*-1.39 (0.72)*	-0.09 (0.25)	0.03 (0.83)	0.14 (0.22)	-0.68 (0.57)	-0.41 (0.47)	-0.23 (0.44)	-0.19 (0.47)	0.14 (0.37)	-0.41 (0.39)	0.17 (0.35)
η_{12}					0.10 (0.24)	1.05 (0.93)	0.29 (0.46)	0.19 (0.42)	0.44 (0.43)	-0.31 (0.38)	0.22 (0.41)	-0.10 (0.49)
η_{13}									0.30 (0.38)	-0.20 (0.39)	0.07 (0.35)	-0.02 (0.46)
α_2	*-0.04 (0.03)*	*-0.30** (0.14)*	0.00 (0.02)	-0.45*** (0.12)	-0.01 (0.02)	-0.39*** (0.16)	-0.03 (0.06)	-0.31*** (0.12)	-0.08** (0.04)	0.08 (0.07)	-0.04 (0.06)	-0.36*** (0.12)
γ_{21}	*-0.02 (0.02)*	*-0.00 (0.04)*	0.08*** (0.03)	0.00 (0.11)	0.02 (0.02)	-0.13 (0.08)	0.00 (0.03)	-0.03 (0.06)	0.04* (0.02)	-0.15*** (0.04)	0.06** (0.03)	-0.09 (0.06)
γ_{22}					0.03 (0.02)	-0.15 (0.10)	0.01 (0.03)	0.01 (0.07)	-0.03 (0.02)	-0.09* (0.05)	-0.03 (0.03)	-0.10 (0.07)
γ_{23}									0.04* (0.02)	-0.05 (0.05)	0.01 (0.03)	-0.01 (0.09)
η_{21}	*0.51*** (0.08)*	*-0.06 (0.18)*	1.12*** (0.06)	-0.11 (0.17)	0.29 (inf)	-0.04 (0.25)	1.08*** (0.15)	-0.30* (0.16)	0.48*** (0.17)	0.10 (0.12)	0.76*** (0.13)	-0.21 (0.17)
η_{22}					0.14*	0.27 (0.37)	-0.12 (0.15)	0.13 (0.16)	0.24*	0.13 (0.13)	0.08 (0.11)	0.07 (0.19)

Parameters/lags			1 month				2 months				3 months	
	$regime_n$	$regime_n$	$regime_r$	$regime_r$	$regime_n$	$regime_n$	$regime_r$	$regime_r$	$regime_n$	$regime_n$	$regime_r$	$regime_r$
					(0.08)				(0.14)			
η_{23}									0.26** (0.12)	0.00 (inf)	0.10 (0.11)	0.10 (0.19)
p_{11}	*0.93****		0.90***		0.92***		0.91***		0.90***		0.90***	
p_{22}		*0.57****		0.87***		0.68***		0.83***		0.89***		0.87***
duration	*14.53*	*2.35*	10.36	7.62	13.15	3.17	10.82	5.94	10.15	8.84	10.34	7.72
Loglik	*-494.07*		-574.85		-515.36		-570.45		-529.87		-570.05	

Note: Data sources, TEJ. Asterisks refer to the level of significance are: * 10%, ** 5%, *** 1%. Loglik is the log-likelihood. The numbers indicated in parenthesis are standard error. $regime_n$ and $regime_r$ represents nominal and real exchange rate respectively.

Table 5

TWD←→JPY Results from Markov switching VAR model

Parameters/lags			1 month				2 months				3 months	
	$regime_n$	$regime_n$	$regime_r$	$regime_r$	$regime_n$	$regime_n$	$regime_r$	$regime_r$	$regime_n$	$regime_n$	$regime_r$	$regime_r$
α_1	-0.01 (0.16)	0.27 (0.21)	-0.06 (0.11)	0.80** (0.34)	-0.02 (0.12)	0.12 (0.26)	-0.10 (0.13)	0.31 (0.25)	*-0.08 (0.13)*	*0.85 (0.63)*	-0.08 (0.12)	0.38 (0.41)
γ_{11}	0.54*** (0.14)	0.14 (0.11)	0.30*** (0.10)	0.09 (0.14)	0.45*** (0.14)	0.24 (0.15)	0.56*** (0.12)	0.28** (0.16)	*0.14* (0.08)*	*-0.02 (0.27)*	0.15*** (0.06)	0.34 (inf)
γ_{12}					-0.06 (0.15)	0.05 (0.21)	-0.14 (0.11)	0.04 (0.12)	*-0.13** (0.07)*	*0.02 (0.04)*	-0.08 (0.07)	0.05 (0.17)
γ_{13}									*0.18*** (0.07)*	*-0.37 (0.21)*	0.19*** (0.08)	-0.17 (0.22)
η_{11}	-0.08 (0.05)	0.06 (0.06)	-0.01 (0.04)	-0.01 (0.09)	-0.10** (0.05)	-0.03 (0.09)	-0.04 (0.04)	-0.05 (0.08)	*-0.06** (0.03)*	*-0.15 (inf)*	-0.03 (0.04)	-0.07 (0.10)
η_{12}					-0.06 (0.05)	0.37 (0.18)	-0.06 (0.05)	0.12 (0.10)	*-0.03 (0.03)*	*0.52 (inf)*	-0.06* (0.03)	0.36** (0.15)
η_{13}									*-0.02 (0.03)*	*-0.22 (0.28)*	0.02 (0.04)	-0.34 (0.14)
α_2	0.21 (0.26)	-0.11 (0.41)	0.08 (0.24)	-1.52** (0.12)	0.10 (0.25)	-0.05 (0.44)	-0.29 (0.26)	0.03 (0.40)	*-0.02 (0.28)*	*0.23 (2.02)*	-0.36 (0.23)	0.06 (0.45)
γ_{21}	0.54** (0.25)	0.25 (0.19)	0.78*** (0.21)	0.24 (0.23)	0.32 (0.22)	-0.04 (0.21)	0.36 (0.22)	-0.13 (0.22)	*0.17 (0.14)*	*0.74 (0.45)*	0.13 (0.15)	-0.01 (inf)
γ_{22}					0.20 (0.29)	-0.05 (0.41)	0.18 (0.24)	-0.24 (0.25)	*0.11 (0.12)*	*-0.52 (0.58)*	0.03 (0.15)	-0.19 (0.43)
γ_{23}									*0.42*** (0.10)*	*-1.28*** (0.02)*	0.39*** (0.15)	-0.23 (0.45)
η_{21}	0.53*** (0.11)	-0.41*** (0.10)	-0.02 (0.09)	-0.27 (0.17)	0.26*** (0.09)	-0.39*** (0.16)	0.29*** (0.09)	-0.51*** (0.14)	*0.06 (0.08)*	*-0.62 (0.61)*	0.12 (0.08)	-0.68*** (0.23)

Parameters/lags			1 month				2 months				3 months	
	$regime_n$	$regime_n$	$regime_r$	$regime_r$	$regime_n$	$regime_n$	$regime_r$	$regime_r$	$regime_n$	$regime_n$	$regime_r$	$regime_r$
η_{22}					0.04 (0.12)	0.22 (0.40)	0.00 (0.04)	0.23 (0.21)	*0.00 (inf)*	*0.64 (0.52)*	-0.01 (0.05)	0.63* (0.38)
η_{23}									*-0.05 (0.05)*	*-0.22 (0.21)*	-0.04 (0.07)	0.30 (0.26)
p_{11}	0.91***		0.92***		0.90***		0.90***		*0.92****		0.90***	
p_{22}		0.89***		0.75***		0.87***		0.87***		*0.74*		0.86***
duration	10.65	7.47	12.13	4.00	10.32	7.82	10.40	7.43	*12.23*	*3.78*	10.44	7.25
Loglik	-986.03		-978.33		-986.01		-988.36		*-963.87*		-973.77	

Note: Data sources, TEJ. Asterisks refer to the level of significance are: * 10%, ** 5%, *** 1%. Loglik is the log-likelihood. The numbers indicated in parenthesis are standard error. $regime_n$ and $regime_r$ represents nominal and real exchange rate respectively.

Table 6

TWD←→KRW Results from Markov switching VAR model

Parameters/lags			1 month				2 months				3 months	
	$regime_n$	$regime_n$	$regime_r$	$regime_r$	$regime_n$	$regime_n$	$regime_r$	$regime_r$	$regime_n$	$regime_n$	$regime_r$	$regime_r$
α_1	-0.09 (0.27)	-0.05 (0.50)	0.33** (0.15)	0.77 (0.28)	*-0.01 (0.13)*	*0.37 (0.24)*	0.02 (0.15)	0.25 (0.34)	0.04 (0.15)	-0.04 (0.21)	0.06 (0.15)	-0.05 (0.22)
γ_{11}	-0.22 (0.17)	0.12 (0.13)	-0.43*** (0.16)	0.18 (0.13)	*-0.08 (0.09)*	*0.22 (0.17)*	-0.16* (0.09)	0.07 (0.19)	-0.01 (0.06)	0.06 (0.14)	-0.00 (0.05)	0.06 (0.19)
γ_{12}					*-0.26*** (0.08)*	*0.31 (0.22)*	-0.22* (0.13)	0.18 (0.28)	-0.08 (0.11)	0.07 (0.15)	-0.07 (0.12)	0.05 (0.35)
γ_{13}									0.15 (0.11)	-0.14 (0.33)	0.15 (0.11)	-0.14 (0.17)
η_{11}	0.29*** (0.11)	0.02 (0.02)	0.14*** (0.03)	-0.05 (0.05)	*0.22*** (0.02)*	*-0.20*** (0.06)*	0.22*** (0.03)	-0.15*** (0.06)	0.38*** (0.07)	0.04 (0.05)	0.41*** (0.06)	0.01 (0.11)
η_{12}					*0.11*** (0.03)*	*-0.23*** (0.08)*	0.13*** (0.03)	-0.22*** (0.08)	0.02 (0.04)	-0.17*** (0.05)	0.03 (0.04)	-0.18* (0.10)
η_{13}									-0.14*** (0.04)	0.08 (0.06)	-0.15** (0.06)	0.06 (0.06)
α_2	0.10 (0.68)	-0.10 (1.39)	-0.27 (0.35)	0.13 (0.73)	*0.24 (0.33)*	*-0.13 (0.69)*	0.12 (0.33)	-0.25 (0.80)	0.14 (0.41)	-0.14 (0.56)	0.01 (0.40)	-0.01 (1.97)
γ_{21}	0.46* (0.27)	-0.28 (0.31)	0.23 (0.29)	0.55* (0.32)	*0.99*** (0.29)*	*-0.10 (0.51)*	0.95*** (0.33)	-0.46 (0.59)	0.42 (0.29)	-0.39 (0.33)	0.37 (0.32)	-0.34 (0.53)
γ_{22}					*-1.05*** (0.30)*	*0.31 (0.56)*	-1.01*** (0.30)	0.11 (0.58)	-0.62** (0.31)	0.58 (0.38)	-0.68 (0.43)	0.64 (0.46)
γ_{23}									0.31 (0.28)	-0.29 (0.36)	0.28 (0.27)	-0.26 (0.59)
η_{21}	0.33*** (0.08)	-0.10 (0.17)	0.55*** (0.10)	-0.69*** (0.08)	*0.22*** (0.08)*	*-0.51*** (0.18)*	0.18** (0.08)	-0.50*** (0.18)	0.02 (0.16)	-0.00 (inf)	0.09 (0.33)	-0.04 (0.22)

Parameters/lags			1 month				2 months				3 months	
	$regime_n$	$regime_n$	$regime_r$	$regime_r$	$regime_n$	$regime_n$	$regime_r$	$regime_r$	$regime_n$	$regime_n$	$regime_r$	$regime_r$
η_{22}					*0.10* *(0.08)*	*-0.66**** *(0.22)*	0.17* (0.09)	-0.66*** (0.21)	0.14 (0.14)	-0.16 (0.20)	0.14 (0.19)	-0.17 (0.45)
η_{23}									-0.01 (0.10)	0.09 (0.20)	-0.07 (0.15)	0.07 (0.31)
p_{11}	0.90***		0.91***		*0.92****		0.92***		0.90***		0.90***	
p_{22}		0.88		0.81***		*0.71****		0.75***		0.90***		0.90***
duration	10.20	8.51	11.05	5.39	*12.03*	*3.40*	11.95	4.05	10.04	9.66	10.04	9.66
Loglik	-1056.81		-1055.95		*-1033.28*		-1035.91		-1051.97		-1059.78	

Note: Data sources, TEJ. Asterisks refer to the level of significance are: * 10%, ** 5%, *** 1%. Loglik is the log-likelihood. The numbers indicated in parenthesis are standard error. $regime_n$ and $regime_r$ represents nominal and real exchange rate respectively.

Figure 1

Regime 1 smoothed probabilities of TWD←→CNY

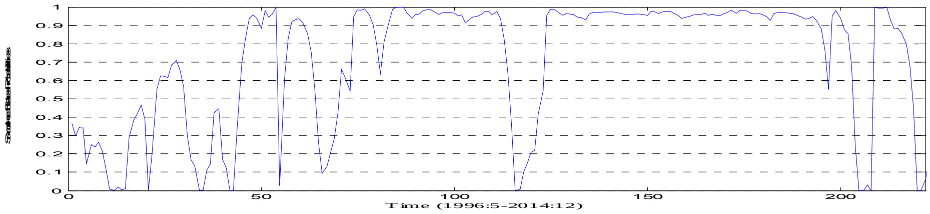


Figure 2

Regime 1 smoothed probabilities of TWD←→JPY

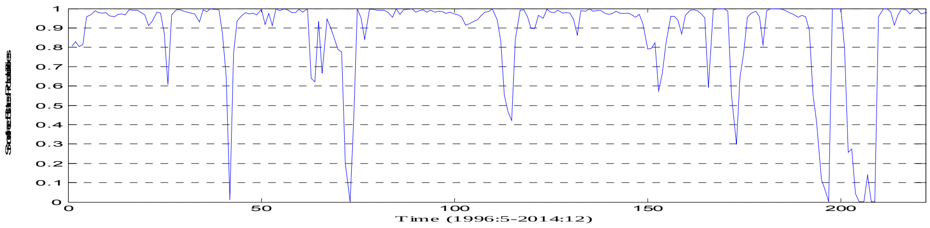
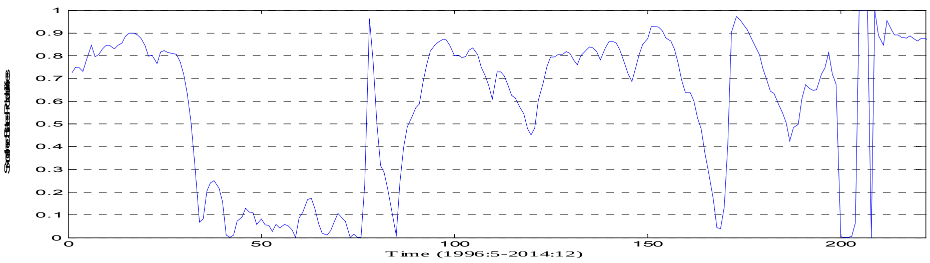


Figure 3

Regime 1 smoothed probabilities of TWD←→KRW



III.4. Mutual influences of multi-currencies

The foreign exchange rate reflects the fundamental factors and government policies between countries. The international finance market includes many country's currencies; they are all mutually affected. In the recent decade, the integrations of regional economies are flourishing. This could bring the result of foreign exchange rates being more affected by regional economic and exchange rates than before.

We illustrate the Granger causalities of two exchange rates in the above discussions. Now, we extend Eqs. (7) and (8) to four exchange rates cases, and present the results of foreign exchange rates affected by multi-currencies.

$$y_{1,t} = \alpha_{1s_t} + \sum_{i=1}^{l_{11}} \gamma_{1i_{s_t}} y_{1,t-i} + \sum_{i=1}^{l_{12}} \eta_{1i_{s_t}} y_{2,t-i} + \sum_{i=1}^{l_{13}} \lambda_{1i_{s_t}} y_{3,t-i} + \sum_{i=1}^{l_{14}} \nu_{1i_{s_t}} y_{4,t-i} + \varepsilon_{1_{s_t}} \tag{9}$$

$$y_{2,t} = \alpha_{2s_t} + \sum_{i=1}^{l_{21}} \gamma_{2i_{s_t}} y_{1,t-i} + \sum_{i=1}^{l_{22}} \eta_{2i_{s_t}} y_{2,t-i} + \sum_{i=1}^{l_{23}} \lambda_{2i_{s_t}} y_{3,t-i} + \sum_{i=1}^{l_{24}} \nu_{2i_{s_t}} y_{4,t-i} + \varepsilon_{2_{s_t}} \tag{10}$$

$$y_{3,t} = \alpha_{3s_t} + \sum_{i=1}^{l_{31}} \gamma_{3i_{s_t}} y_{1,t-i} + \sum_{i=1}^{l_{32}} \eta_{3i_{s_t}} y_{2,t-i} + \sum_{i=1}^{l_{33}} \lambda_{3i_{s_t}} y_{3,t-i} + \sum_{i=1}^{l_{34}} \nu_{3i_{s_t}} y_{4,t-i} + \varepsilon_{3_{s_t}} \tag{11}$$

$$y_{4,t} = \alpha_{4s_t} + \sum_{i=1}^{l_{41}} \gamma_{4i_{s_t}} y_{1,t-i} + \sum_{i=1}^{l_{42}} \eta_{4i_{s_t}} y_{2,t-i} + \sum_{i=1}^{l_{43}} \lambda_{4i_{s_t}} y_{3,t-i} + \sum_{i=1}^{l_{44}} \nu_{4i_{s_t}} y_{4,t-i} + \varepsilon_{1_{s_t}} \tag{12}$$

Table 7 displays the estimated results; we only present the parameters related to the TWD and skip the other exchange rates for simplification. Under the criterion of minimum log likelihood, the optimal lag period is one month for the nominal exchange rate series; the others are just slightly higher than it. For example, at the three months nominal exchange rate case, the log likelihood is -1733.81 compared to one month's data -1732.95. The benefit of using three months data is including more information that we are interested in.

In this sub-section, we use multi-currencies to investigate the affections among exchange rates. The multi-currencies cases bring a more complicated environment which would be close to the real international financial markets. This study reveals the mutual effects of exchange rates. Take the nominal exchange rate lag for three months for example: the TWD Granger causes CNY, JPY and KRW at different periods; for CNY and KRW, Granger causes TWD to move in the opposite direction. This is not the same as linear model performance.

Table 7

TWD←→CNY+JPY+KRW Results from Markov switching VAR model (only related with TWD)

Parameters/lags			1 month				2 months				3 months	
	$regime_n$	$regime_n$	$regime_r$	$regime_r$	$regime_n$	$regime_n$	$regime_r$	$regime_r$	$regime_n$	$regime_n$	$regime_r$	$regime_r$
η_{11}	0.16 (0.37)	-0.06 (0.41)	-0.10 (0.20)	0.03 (inf)	-0.02 (0.32)	0.04 (0.15)	-0.47 (0.32)	0.34 (0.51)	-0.13 (0.31)	0.14 (0.27)	-0.44 (0.32)	0.19 (0.59)
η_{12}					0.58** (0.29)	-0.45 (0.67)	0.20 (0.28)	-0.20 (0.62)	0.52** (0.24)	-0.49 (0.40)	0.27 (0.32)	-0.23 (0.66)
η_{13}									0.21 (0.24)	-0.18 (inf)	-0.04 (0.38)	0.02 (inf)
λ_{11}	0.00 (0.01)	-0.09** (0.05)	-0.13*** (0.04)	0.05 (0.09)	-0.04 (0.04)	0.30** (0.14)	-0.05 (0.04)	0.18 (0.12)	-0.01*** (0.03)	0.10 (0.12)	-0.11*** (0.04)	0.20 (0.16)
λ_{12}					0.06* (0.04)	0.18 (0.19)	0.06 (0.04)	-0.04 (0.14)	0.03 (0.03)	0.25* (0.13)	0.05 (0.04)	0.27*** (0.09)
λ_{13}									0.08** (0.04)	-0.13* (0.07)	0.06 (0.04)	-0.01 (0.18)
ν_{11}	0.17 (0.03)	0.02*** (0.05)	0.28*** (0.03)	-0.16*** (0.06)	0.26*** (0.03)	-0.29*** (0.10)	0.21*** (0.03)	-0.19** (0.09)	0.26*** (0.03)	-0.15*** (0.05)	0.26*** (0.03)	-0.22** (0.09)
ν_{12}					0.12*** (0.03)	-0.22** (0.10)	0.07*** (0.03)	-0.31** (0.13)	0.05* (0.03)	-0.18 (0.11)	0.04 (0.03)	-0.34*** (0.11)
ν_{13}									-0.08*** (0.03)	0.06* (0.03)	-0.08*** (0.03)	0.11 (0.12)
γ_{21}	0.04* (0.02)	-0.03 (0.02)	-0.00 (inf)	0.02 (inf)	-0.01 (0.03)	-0.12 (0.08)	-0.04 (0.04)	0.07 (0.20)	0.03 (0.03)	-0.27*** (0.11)	-0.01 (0.04)	-0.04 (0.12)
γ_{22}					0.01 (0.02)	-0.13 (0.20)	-0.04 (0.03)	0.14 (0.24)	0.01 (0.02	-0.23** (0.11)	-0.01 (0.04)	0.03 (0.15)

Parameters/lags			1 month				2 months				3 months	
	$regime_n$	$regime_n$	$regime_r$	$regime_r$	$regime_n$	$regime_n$	$regime_r$	$regime_r$	$regime_n$	$regime_n$	$regime_r$	$regime_r$
γ_{23}									0.02 (0.02	0.03 (0.08)	-0.02 (0.15)	0.22 (0.15)
γ_{31}	0.16 (0.16)	-0.07 (0.21)	0.23 (0.17)	-0.04 (inf)	0.18 (0.16)	-0.15 (0.46)	0.16 (0.16)	-0.14 (0.83)	0.15 (0.16)	-0.12 (0.39)	0.30* (0.16)	-0.17 (0.44)
γ_{32}					0.00 (0.05)	-0.02 (0.29)	-0.02 (0.18)	0.02 (inf)	-0.06 (0.09)	0.05 (0.69)	-0.13 (0.17)	0.01 (inf)
γ_{33}									0.18 (0.14)	-0.13 (0.62)	0.38 (0.17)	-0.17 (0.17)
γ_{41}	0.53 (inf)	-0.42 (0.33)	0.45 (0.25)	-0.24 (0.44)	0.70 (0.24)	-0.62 (0.77)	0.57** (0.23)	-0.52 (1.09)	0.58*** (0.19)	-0.49 (0.63)	0.71*** (0.24)	-0.46 (0.66)
γ_{42}					-0.66*** (0.23)	0.40 (0.98)	-0.60*** (0.24)	0.49 (0.88)	-0.60*** (0.21)	0.54 (1.02)	-0.76*** (0.24)	0.54 (0.73)
γ_{43}									0.27 (0.21)	-0.21 (0.50	0.34 (0.26)	-0.13 (1.01)
p_{11}	0.90		0.90***		0.90***		0.90***		0.90***		0.90***	
p_{22}		0.89		0.88		0.89***		0.89***		0.90		0.88***
duration	10.12	9.04	10.27	8.11	10.12	9.04	10.08	9.35	10.04	9.62	10.19	8.53
Loglik	-1732.95		-1770.23		-1741.04		-1769.74		-1733.81		-1758.03	

Note: Data sources, TEJ. Asterisks refer to the level of significance are: * 10%, ** 5%, *** 1%. Loglik is the log-likelihood. The numbers indicated in parenthesis are standard error. $regime_n$ and $regime_r$ represents nominal and real exchange rate respectively.

III.5. Dynamic relationships of foreign exchange rates

The information from the above investigations is stated as follows:

a) The returns of foreign exchange rates are not linear. Tests of the BDS and Markov-switching VAR models show that the nonlinear model is more suitable to describe the dynamics relationships of foreign exchange rates, which is in line with many previous studies (Engle & Hamilton, 1990; LeBaron, 1992; Bekaert & Gray, 1996 and Engle & Hakkio, 1996).

b) Markov-switching Granger causalities differ remarkably from the linear model. Markov-switching Granger causalities are varied with respect to the sample lagged periods; linear Granger causalities are relatively persistent.

Taking TWD←→CNY and TWD←→JPY for example, the linear model shows that for the TWD, it does not Granger cause CNY and JPY; for the CNY and JPY, it does not Granger causes TWD at each lag month. However, we found many Granger causalities between exchange rates based on the Markov-switching VAR model. TWD←→KRW has a similar phenomenon. This indicates that the Markov-switching VAR model provides more accurate information among exchange rates' causality. Since the Markov-switching Granger causalities are varied with respect to the sample lagged period. When considering this with the findings of previous studies, we may conclude of the inconstant nature of Granger causality. Table 8 illustrates the comparisons of linear and Markov-switching Granger causalities of three months lag case.

Table 8

Comparisons of linear and Markov-switching Granger causality (3 months lag)

Granger causality	linear		nonlinear			
	n	r	$regime_{n1}$	$regime_{n2}$	$regime_{r1}$	$regime_{r2}$
TWD→CNY						
CNY→TWD			$\gamma_{21}^{*}, \gamma_{31}^{*},$	$\gamma_{21}^{***}, \gamma_{22}^{*}$	γ_{21}^{**}	
TWD→JPY			η_{11}^{**}		η_{12}^{*}	η_{12}^{**}
JPY→TWD			γ_{23}^{***}	γ_{23}^{***}	γ_{23}^{***}	
TWD→KRW	***	**	$\eta_{11}^{***}, \eta_{13}^{***}$	η_{12}^{***}	$\eta_{11}^{***}, \eta_{13}^{**}$	η_{12}^{*}
KRW→TWD	*	**	γ_{22}^{**}			

Note: Asterisks refer to the level of significance are: * 10%, ** 5%, *** 1%. n, $regime_n$ and r $regime_r$ represent nominal and real exchange rate respectively.

c) The empirical findings on the influence of interest rate are mixed. This paper uses nominal and real exchange rates to test the Granger causality between exchange rates. The linear Granger causality of both exchange rates exhibits the similar results, except the KRW_n →TWD_n and KRW_r →TWD_r. In the Markov-switching VAR model, the nominal exchange rate always shows the lowest log likelihood. This result may not be surprising as recent studies on exchange rates could not support the influences of interest rate (Wu, 2015). The empirical findings on the influence of interest rates are mixed, although market participants pay a lot of attention to it.

d) The multi-currencies model is a better approximation of international financial markets. We explore the Markov-switching Granger causality of several Asia-Pacific exchange rates, and find plentiful causalities between exchange rates. Maybe we should declare that it takes time for the exchange rate adjustment. The mutual relationships between Asia-Pacific exchange rates are profound and lasting.

IV. Conclusion

Granger causality is one of the most popular methodologies with which to explore the relationships between variables. Causal information can always be shown to be useful, and can produce better forecasts. Conventional studies, frequently based on the linear assumption of the time series, obtained rich achievements.

However, the returns of foreign exchange rates have been proven to be nonlinear. This paper adopts the Markov-switching VAR model to explore the nonlinear Granger causality between exchange rates. Four major Asia-Pacific exchange rates were employed: the CNY, JPY, KRW and TWD. This paper focuses on the cross affections between the TWD and the other exchange rates, and then presents the empirical results.

The results show that the Markov-switching Granger causalities are varied with respect to the sample lagged periods. They differ remarkably from the linear model. This suggests that we should use the Markov-switching VAR model to recheck the relationship between exchange rates, and capture the fluctuations of Asia-Pacific exchange rates.

This paper uses only four major Asia-Pacific exchange rates; this may cause some limitations in regard to extending our results. We chose these four exchange rates based on the heavy trade between countries and higher correlation between the exchange rates, so the effect may not be serious. We still could extend our findings and try to apply them to real situations.

References

Bekaert, G. and Gray, S.F., 1996. Target zones and exchange rate: an empirical investigation. *NBER working paper*, no. 5445.

Boudjellaba, H., Dufour, J.M. and Roy, R., 1994. Simplified conditions for noncausality between vectors in multivariate ARMA models. *Journal of Econometrics*, 63(1), pp. 271-287.

Brock, W., Dechert, W. and Scheinkman, J., 1987. *A test for independence based on the correlation dimension*. University of Wisconsin, Madison, University of Houston, and University of Chicago.

Christopoulos, D.K. and $Le\acute{o}n$-Ledesma, M.A., 2008. Testing for Granger (non-)causality in a time-varying coefficient VAR model. *Journal of Forecasting*, 27(4), pp. 293-303.

Droumaguet, M., Warne, A. and $Wo\acute{z}niak$, T., 2015. Granger causality and regime inference in Bayesian Markov-switching VARs. *European Central Bank, Working Paper Series*, no. 1794.

Engle, C. and Hakkio, C.S., 1996. The distribution of exchange rates in the EMS. *International Journal of Finance and Economic,* 1(1), pp. 55-67.

Engel, C. and Hamilton, J.D., 1990. Long swings in the dollar: are they in the data and do markets know it? *American Economic Review,* 80(4), pp. 689-713.

Granger, C.W.J., 1969. Investigating causal relations by econometric models and cross-spectral methods. *Econometrica,* 37(3), pp. 424-438.

Hamilton, J.D., 1989. A new approach to the economic analysis of nonstationary time series and business cycle. *Econometrica,* 57(2), pp. 357-384.

Hamilton, J.D., 1994. *Time Series Analysis.* Princeton, N.J., Princeton University Press.

Hamilton, J.D. and Susmel, R., 1994. Autoregressive conditional heteroscedasticity and changes in regime. *Journal of Econometrics,* 64(1-2), pp. 307-333.

Krolzig, H.M., 1997. *Markov switching vector autoregressions. Modelling, statistics inference and application to business cycle analysis,* Berlin, Springer.

Krolzig, H.M. 2000. *Predicting Markov-switching vector autoregressive processes.* Unpublished manuscript, Nuffield College, University of Oxford.

LeBaron, B., 1992. Some relations between volatility and serial correlations in stock market returns. *The Journal of Business,* 65(2), pp. 199-219.

Otranto, E., 2005. The multi-chain Markov switching model. *Journal of Forecasting*, 24(7), pp. 523-537.

Psaradakis, Z. Ravn, M.O. and Sola, M., 2005. Markov switching causality and the money-output relationship. *Journal of Applied Econometrics,* 20(5), pp. 665-683.

Teräsvirta, T., 1994. Specification, estimation and evaluation of smooth transition autoregressive models. *Journal of the American Statistical Association,* 89(425), pp. 208-218.

Thoma, M.A., 1994. Subsample instability and asymmetries in money- income causality. *Journal of Econometrics,* 64(1-2), pp. 279-306.

Tong, H., 1978. *On a threshold model.* In: Chen, C, (ed.) *Pattern Recognition and Signal Processing*. Amsterdam, Sijthoff & Noordhoff.

Tong, H. and Lim, K.S., 1980. Threshold autoregressions, limit cycles, and data. *Journal of the Royal Statistical Society,* 42(3), pp. 245-292.

$Wo\acute{z}niak$, T., 2012. Testing causality between two vectors in multivariate GARCH models. *EUI Working Papers ECO 2012/20*, European University Institute, Florence, Italy.

Wu, J.T., 2015. Markov regimes switching with monetary fundamental-based exchange rate model. *Asia Pacific Management Review*, 20(2), pp. 79-89.