# 6. NONLINEAR BEHAVIOR OF THE US STOCK PRICE-DIVIDEND: EVIDENCE FROM THRESHOLD UNIT ROOT TESTS[1]

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## Abstract

*This study investigates the behavior of US stock price–dividend relationships over the period 1871:01 to 2012:03 using a two-regime Threshold Autoregressive (TAR) model with an autoregressive unit root developed by Caner and Hansen (2001), which allows for simultaneously testing nonlinearity and non-stationary. Our findings indicate that the US stock price-dividend is a nonlinear series that is characterized by a unit root process in a particular month; the stock price-dividend ratio shows a decrease by more than 7.17% between the previous year and the previous fourth month.*

**Keywords:** threshold autoregressive (TAR); US stock price-dividend; regime change
**JEL Classification**: C32, C53, G14

## 1. Introduction

Stock market efficiency is among the most popular research topic in the international financial literature. One of the most actively investigated financial phenomena of the last decade has been the behavior of aggregate US stock prices. Stock market efficiency implies that prices respond quickly and accurately to relevant information. Over the last decades, empirical research on the behavior of stock prices has been unprecedented since the first studies by Fama and French (1998), Lo and MacKinlay (1998), and Poterba and Summers (1998). The testing for mean reversion in stock prices, i.e., whether or not stock prices are characterized by a unit root, has gained momentum as it has implications for the efficient market hypothesis, which is based on the premise that stock market results are unpredictable from the previous price changes. If the efficient market hypothesis holds, then stock prices should be characterized by a unit root. Another importance of testing for mean reversion in stock

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prices is that if a unit root process is found, then this implies that volatility in stock markets will increase in the long run without limits, which has implications for investment decisions and strategies.

There is a large body of the literature that investigates the efficient market hypothesis using a variety of methodologies, with mixed results. Many studies have found that indexes are not characterized by a unit root ( Urrutia, 1995; Lo and MacKinlay, 1988; Poterba and Summers, 1988; Grieb and Reyes, 1999; Chaudhuri and Wu, 2003; Shively, 2003; Narayan, 2008), while others have found stock indexes to be a unit root process (Choudhry, 1997; Huber, 1997; Liu *et al*., 1997; Kawakatsu, 1999; Narayan and Smyth, 2004, 2005, 2007; Narayan, 2005, 2006; Ozdemir, 2008).

Additionally, using the influence of the linear Present Value (PV) model to explain the behavior of aggregate US stock prices has also been actively investigated. The linear PV model is more tractable than its nonlinear version, and this accounts for its use in empirical work. Froot and Obstfeld (1991) proposed a standard PV model with intrinsic bubbles, where speculation by rational investors would create threshold effects in the stock price–dividend relation. As discussed by Campbell *et al*. (1997), the linear PV model is based on the assumption of constant expected stock returns. This assumption is conveniently simple, but it contradicts the empirical evidence discussed by Campbell *et al*. (1997) supporting the predictability and time-variation of expected stock returns. They illustrate that when expected stock returns are time-varying, the correct PV formula is nonlinear.

Recently, empirical studies that investigate the presence of nonlinearities in the stock price-dividend relation include the study by Kanas (2003), who provides some empirical evidence of nonlinearities in the PV model using US annual data for 1871–1999 and follows the procedure for nonlinear cointegration suggested by Granger and Hallman (1991). Later, Kanas (2005) used monthly data for the period 1978:1 to 2002:5 and three nonlinear nonparametric techniques, obtaining evidence on the existence of nonlinearities in the stock price-dividend relation for the United Kingdom, the United States, Japan, and Germany.

In sum, according to several empirical studies, the linear PV model fails to explain the behavior of stock prices in the long run (e.g, Bohl and Siklos, 2004; Caporale and Gil-Alana, 2004; Koustas and Serletis, 2005; Cuñado *et al*., 2005; Kanas, 2005; and the references therein). The present study examines whether this failure of the linear PV model can be attributed to nonlinearities in the stock price-dividend relation.

This study empirically tests whether there have been nonlinearities in the stock price-dividend relation for the US market. The data is annual, covering the years 1871:01 to 2012:03. We employ the Threshold Autoregressive (TAR) models that allow endogenously derived threshold effects in the evolution of the US stock price-dividend ratio. Nonlinearity is tested using the technique developed by Caner and Hansen (2001) for a threshold whose location is unknown a priori. Hence, a mean-reverting dynamic behavior of the US stock price-dividend ratio should be expected once such a threshold is reached.

The study is organized as follows: data is presented and summarized in Section II, econometric methodology is outlined in Section III, empirical results are given in Section IV, and conclusions are drawn in Section V.

## 2. Data

We analyze the US Standard and Poor's 500 stock price index and dividend data over the period 1871:01 to 2012:03; the data was collected from Professor Shiller's Website.[4] The stock price index is the January values of the Standard & Poor's 500 Composite Stock Price index; the evolution of the real stock price-dividend[5] ratio is shown in Figure 1.

## 3. Econometric Methodology

### 3.1 Caner and Hansen (2001) Two-Regime TAR Model

Hansen (1996, 1997, and 2000) and Caner and Hansen (2001) present some new results for the TAR model introduced by Tong (1978, 1983, and 1990). In particular, they develop new tests for threshold effects, estimate the threshold parameter, and construct asymptotic confidence intervals for the threshold parameter.

More specifically, they consider a two-regime TAR (k) model with an autoregressive unit root and two regimes, $\theta_1$ and $\theta_2$:

$$\Delta y_t = \theta_1 x_{t-1} I(Z_{t-1} < \lambda) + \theta_2 x_{t-1} I(Z_{t-1} \geq \lambda) + \varepsilon_t \qquad (1)$$

with

$$x_{t-1} = (y_{t-1}, 1, \Delta y_{t-1}, \ldots, \Delta y_{t-k})', \qquad (2)$$

where: y is the logarithm of the US stock price-dividend index for t = 1, 2, . . . , T; $\varepsilon_t$ is an i.i.d. error; I (expression) is the indicator function that equals to 1 if the expression in the parentheses is true and 0 otherwise; $Z_t = y_t - y_{t-m}$ for some $m \geq 1$ is the threshold variable; and $k \geq 1$ is the autoregressive order.

The variable $Z_t$ has clear financial meaning when acting as return at the time horizon of m months. The threshold parameter $\lambda$ is unknown and represents the level of the variable $y_t$ that triggers a regime change, if any. The components of $\theta_1$ and $\theta_2$ can be partitioned as follows:

$$\theta_1 = \begin{pmatrix} \rho_1 \\ \beta_1 \\ \alpha_1 \end{pmatrix} \text{ and } \theta_2 = \begin{pmatrix} \rho_2 \\ \beta_2 \\ \alpha_2 \end{pmatrix}, \qquad (3)$$

[4]. The data is from Professor Shiller's Web site http://aida.econ.yale.edu/~shiller.

[5]. The monthly dividend data is computed from the S&P four-quarter totals for the quarter since 1926, with linear interpolation to monthly figures. Dividend data before 1926 are from Cowles and associates, interpolated from annual data. Stock price data are monthly averages of daily closing prices through January 2000.

where: $\rho_1$ and $\rho_2$ are slope coefficients on $y_{t-1}$, $\beta_1$ and $\beta_2$ are scalar intercepts, and $\alpha_1$ and $\alpha_2$ are 1 × *k* vectors containing the slope coefficients on the dynamic regressors $(\Delta y_{t-1},..., \Delta y_{t-k})$ in the two regimes.

In order to calibrate the TAR model, the concentrated least squares approach is usually used. The regression procedure is carried out for each value of *m*. The value of $\lambda$ is taken from a compact interval, [$\lambda_1$, $\lambda_2$], in which $\lambda_1$ and $\lambda_2$ are determined by the following constraints:

$$\begin{cases} \Pr(Z_t \le \lambda_1) = \pi_1 \\ \Pr(Z_t \le \lambda_2) = \pi_2 \end{cases}, \tag{4}$$

where: 0 < $\pi_1$ < $\pi_2$ < 1 and $\pi_1$ + $\pi_2$ = 1.

In this work, we impose $\pi_1$ = 0.15 by Basci and Caner (2005). For each $\lambda \in [\lambda_1, \lambda_2]$, the parameters $\rho$ , $\beta$ , and $\alpha$ are estimated by minimizing the objective function

$$Q(\lambda \le \mathrm{m}) = \sum_{t=1}^{T} \in_t (\lambda, m)^2. \tag{5}$$

Let $\in_t (\lambda, m)$ represent the residual from the ordinary least squares for the given $\lambda$ and $m$ . Then, the least squares estimate of the threshold parameter is given by

$$\hat{\lambda} = \min_{\lambda \in [\lambda_1, \lambda_2]} \hat{Q}(\lambda, m). \tag{6}$$

When estimating the TAR model in Equation (1), the two central issues are whether or not there is a threshold effect, and whether the process $y_t$ (price-dividend index) is stationary or not. In this study, standard Wald test statistics, $W = W(\hat{\lambda}) = \sup_{\lambda \in [\lambda_1, \lambda_2]} W(\lambda)$, proposed by Caner and Hansen (2001) are used to test the null hypothesis of the no threshold effect (i.e., the process is linear) of $H_0 : \theta_1 = \theta_2$ , against the alternative of the threshold effect (i.e., the process is nonlinear). If the null hypothesis cannot be rejected, there is no threshold effect, in which case the two vectors of coefficients are identical between the two regimes ($\theta_1 = \theta_2$). They find that $W$ has a non-standard asymptotic null distribution with critical values that cannot be tabulated. Hence, they propose a bootstrap method to compute asymptotic critical values and *p*-values.

### *3.2 The Threshold Unit Root Test*

When there are two regimes delimited by a threshold, there are two parameters, $\rho_1$ and $\rho_2$ , controlling the stationarity of the process $y_t$ . The null hypothesis is as follows:

$$H_0 : \rho_1 = \rho_2 = 0. \tag{7}$$

When the null hypothesis $H_0$ holds, the process $y_t$ has a unit root, and the TAR model (1) can be expressed in terms of the stationary difference, $\Delta y_t$. An alternative hypothesis to the null $H_0$ is as follows:

$$H_1 : \rho_1 < 0 \text{ and } \rho_2 < 0. \quad (8)$$

When $H_1$ holds, the process $y_t$ is stationary and ergodic in both regimes. Another alternative deals with a partial unit root, which is expressed as follows:

$$H_2 : \begin{cases} \rho_1 < 0 & and \quad \rho_2 = 0, \\ \rho_1 = 0 & and \quad \rho_2 < 0. \end{cases} \text{ or} \quad (9)$$

When H2 holds, the process $y_t$ has a unit root in one regime and is stationary in the other, showing mean reversion behavior.

The null hypothesis is tested against the unrestricted alternative, $\rho_1 \neq 0$ or $\rho_2 \neq 0$, using Wald statistics, and is expressed as $R_2 = t_1^2 + t_{21}^2$, where $t_1$ and $t_2$ are the *t*-ratios for $\hat{\rho}_1$ and $\hat{\rho}_2$, respectively, from the OLS estimation. However, Caner and Hansen (2001) note that this two-sided Wald statistic may have less power than a one-sided version of the test. As a result, they recommend the following one-sided Wald statistic:

$$R_1 = t_1^2 I(\hat{\rho}_1 < 0) + t_2^2 I(\hat{\rho}_2 < 0) \quad (10)$$

that tests $H_0$ against the one-sided alternative, $\rho_1 < 0$ or $\rho_2 < 0$. A statistically significant $R_1$ justifies rejecting unit roots in favor of stationarity. However, it does not allow us to discriminate between the stationary case, $H_1$, and the partial unit root case, $H_2$. This requires further examination of the individual $t$ statistics, $t_1$ and $t_2$. Only one of $-t_1$ or $-t_2$ being significant would be consistent with the partial unit root case.

## 4. Empirical Results

In this section, we analyze the possible presence of nonlinearities in the US stock price-dividend ratio over the period 1871:01 to 2012:03 using the methodology presented in the previous section. In the first step of the analysis we perform conventional unit root tests of the monthly stock price-dividend ratio without taking into account possible nonlinearity. The Augmented Dickey–Fuller (ADF), Phillips-Perron (PP), and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests with and without a trend function are reported in Table 1. We test for the unit root in both the logarithm of the US stock price-dividend ratio and its first-order difference, $\Delta y_t$. For the ADF and PP tests, the null hypothesis is that $y_t$ has a unit root, which uses the t-statistic. In contrast, the null of the KPSS method is the stationary of the variable and uses the

LM-statistic. All tests indicate that the US stock price-dividend ratio is a unit root process, while its first-order difference is stationary.

We use the Wald test to examine whether we can reject the linear autoregressive model in favor of a threshold model. In our model, we adopt k = 12. In Table 2, we report the results of the Wald test. Also listed are the bootstrap critical values at three conventional levels, 10%, 5%, and 1%, and the bootstrap *p*-values for threshold variables of the form $Z_t = y_t - y_{t-m}$ for different delay parameters, *m*, ranging from 1 to 12. The bootstrapping is carried out using 5,000 and 10,000 replications. The results are qualitatively the same for both cases, so we report the results using 5,000 replications. For all, *m*, the null hypothesis $\theta_1 = \theta_2$ of linearity is rejected at the significance level of 1%. In other words, the presence of a threshold effect in the monthly US stock price-dividend ratio is statistically significant with a 99% confidence level. According to these results, the linear AR model can be rejected in favor of the TAR model.

The optimal value of decay, *m*, can be determined exogenously, which maximizes the value of W. According to Table 2, the Wald statistic is maximized (W = 112.853) when *m* = 4. Hence, we take $\hat{m}$ = 4 as the optimal decay parameter, which results in a preferred TAR model. Accordingly, the point estimate $\hat{\lambda}$ of the threshold is determined to be −0.0717. Therefore, in this case, for the preferred specification of $\hat{m}$ = 4, we report the least squares parameter estimates $\hat{\theta}_1$ and $\hat{\theta}_2$ with standard errors of the TAR model in Table 3.

The TAR model identifies two regimes depending on whether the variable $Z_t = y_t - y_{t-4}$ lies above or below the threshold $\hat{\lambda}$ = −0.0717. The first regime is when $y_{t-1} - y_{t-4} \leq -0.0717$, which occurs when the US stock price-dividend ratio has fallen cumulatively more than 7.17% in the last four months. About 5% of the observations fall into this first regime. The second regime is when $y_{t-1} - y_{t-4} \geq -0.0717$, which is constituted by all those observations that occur when the *m*-month price variation is no less than −0.0717. Approximately 95% of the observations belong to the second regime. Figure 2 shows the estimated division of the monthly US stock price-dividend ratio into two regimes.

Next, we examine the threshold unit root properties of the US stock price-dividend ratio that possess significant threshold effects. We first compute the one-sided and two-sided threshold unit root test statistics, $R_1$ and $R_2$, respectively, together with the bootstrap critical values at three significance levels, 10%, 5%, and 1%, and the p-values for each delay parameter *m* ranging from 1 to 12. The critical Wald-statistics at three significance levels, as well as the p-values, are calculated according to a bootstrap approach using 5,000 replications. The results are reported in Table 4. The one-sided Wald tests in the left panel show that the statistic $R_1$ is less than 1%. For our preferred model, *m* = 4, the $W$ test statistic (10.10) is smaller than the bootstrap critical value (12.18) at the 1% level of significance. A similar situation is found for the two-sided Wald tests presented in the right panel. Thus, for all *m*, both $R_1$ and $R_2$ are lower than the critical value at the 1% level of significance. These results suggest that the null hypothesis of the presence of a unit root in the monthly US stock price-dividend ratio cannot be rejected at the 1% level of significance.

Although both tests $R_1$ and $R_2$ tests are unable reject the unit root hypothesis, they are not able to discriminate between the full unit root case in both regimes and the partial unit root case in one regime. Thus, we test the partial unit root in the monthly US stock price-dividend ratio by calculating the individual t statistics, $t_1$ and $t_2$. The results are reported in Table 5. The critical Wald-statistics at three significance levels, as well as the *p*-values, are calculated according to a bootstrap approach with 5,000 replications. For our preferred model, *m* = 4, the $t_1$ statistic (2.37) is smaller than the bootstrap critical value (2.53) at the 5% level of significance. We find that, for all *m*, both $t_1$ and $t_2$ are lower than the critical value at the 5% significance level. Hence, we are again unable to reject the unit root null hypothesis in both regimes of the monthly US stock price-dividend ratio. It is noteworthy that same conclusions are reached when we use the asymptotic p-valued tabulated by Caner and Hansen (2001) in the above statistical test.

## 5. Conclusion

In summary, we have adopted the econometric approach of TAR with a unit root developed by Caner and Hansen (2001) to analyze the monthly data of the US stock price-dividend ratio. The TAR model is applied to data for the US stock price-dividend ratio over the period 1871:01 to 2012:03. Two important results emerge from our empirical analysis. First, we find that the US stock price-dividend ratio is found to have a threshold effect of $\hat{\lambda}$ = −0.0717 with strong evidence. In addition, both regimes with the index variation below or above the threshold have significant unit roots, as does the whole time series. Our results indicate that the US stock market exhibits nonlinear behaviors with a unit root.

An important question arises about what we can further learn from the fact that the stock market is nonlinear with a threshold. The presence of a threshold, $\hat{\lambda}$ = −0.0717, means that the market behaves differently when it falls by more than 7.17% in four months. This threshold effect has a direct connection with the concept of large drawdowns in the sense of coarse graining in time for the former and price variation for the latter, which are usually outliers. By scanning different time scales, one might be able to provide evidence for such a connection.

Second, the TAR model that has allowed us to derive endogenously threshold effects in the evolution of the US stock price-dividend relation could explain the changes in the trigger stock prices selling strategies which are followed by private investors participating in portfolio insurance schemes. More specifically, we should expect a mean-reverting dynamic behavior in the US stock price-dividend ratio once such a threshold is reached.

## References


Bohl, M.T. and Siklos, P.L., 2004. The present value model of US stock prices redux: a new testing strategy and some evidence. *The Quarterly Review of Economics and Finance*, 44, pp. 208-23.

Campbell, J.Y. Lo, A.W and MacKinlay, A.C., 1997. *The Econometrics of Financial Markets*. Princeton University Press, Princeton, NJ.

Caner, M. and Hansen, B., 2001. Threshold autoregression with a unit root. *Econometrica*, 69, pp. 1555-1596.

Caporale, G.M., and Gil-Alana, L.A., 2004. Fractional cointegration and tests of present value models. *Review of Financial Economics*, 13, pp. 245-258.

Chaudhuri, K., and Y. Wu, 2003. Random walk versus breaking trend in stock prices: evidence from emerging markets. *Journal of Banking and Finance*, 27, pp. 575-92.

Choudhry, K., 1997. Stochastic trends in stock prices: Evidence from Latin American markets, *Journal of Macroeconomics*, 19, pp. 285-304.

Cuñado, J.L. Gil-Alana, A. and F. Perez de Gracia, 2005. A test for rational bubbles in the NASDAQ stock index: A fractionally integrated approach. *Journal of Banking and Finance*, 29, pp. 2633-2654.

Fama, E.F. and K.R. French, 1988. Dividend yields and expected stock returns. *Journal of Financial Economics*, 22, pp. 3-25.

Froot, K.A. and M. Obstfeld, 1991. Intrinsic bubbles: the case of stock prices. *American Economic Review*, 81, pp. 1189-214.

Granger, C.W. and J.J. J.and Hallman, 1991. Long-memory series with attractors, *Oxford Bulletin of Economics and Statistics*, 53, pp. 11-26.

Grieb, T.A. and M.G. Reyes, 1999. Random walk tests for Latin American equity indexes and individual firms. *Journal of Financial Research*, 22, pp. 371-383.

Hansen, B.E., 1996.Inference when a nuisance parameter is not identified under the null hypothesis. *Econometrica*, 64, pp. 413-430.

Hansen, B.E., 1997. Inference in TAR models. *Studies in Nonlinear Dynamics and Econometrics*, 1, pp. 119–31.

Hansen, B.E., 2000. Sample splitting and threshold estimation. *Econometrica*, 68, pp. 575-603.

Huber, P., 1997. Stock market returns in thin markets: evidence from the Vienna stock exchange. *Applied Financial Economics*, 7, pp. 493-498.

Kanas, A., 2003. Non-linear cointegration between stock prices and dividends. *Applied Economic Letters*, 10, pp. 401-405.

Kanas, A. and M. Genius, 2005. Regime non-stationarity in the US/UK real exchange rate. *Economics Letters*, 87, pp. 407-413.

Kawakatsu, K., and M. R. Morey, 1999. An empirical examination of financial liberalization and the efficiency of the emerging market stock prices. *Journal of Financial Research*, 22, pp. 358-411.

Koustas, Z. and A. Serletis, 2005. Rational bubbles or persistent deviations from market fundamentals. *Journal of Banking and Finance*, 29, pp. 2523-2539.

Lo, A.W. and A. C. MacKinlay, 1988. Stock market prices do not follow random walks: evidence from a simple specification test. *Review of Financial Studies*, 1, pp. 41-66.

Liu, C. Y., S. H. Song., and P. Romilley, 1997. Are Chinese stock markets efficient? A cointegration and causality analysis. *Applied Economic Letters*, 4, pp. 511-515.

Narayan, P.K., 2005. Are the Australian and New Zealand stock prices nonlinear with a unit root? *Applied Economics,* 37, pp. 2161-2166.

Narayan, P.K., 2008. Do shocks to G7 stock prices have a permanent effect? Evidence from panel unit root tests with structural change. *Mathematics and Computers in Simulation*, 77, pp. 369-373.

Narayan, P.K. and R. Smyth, 2004. Is South Korea’s stock market efficient? *Applied Economics Letters* 11, pp. 707-710.

Narayan, P. K. and R. Smyth, 2005. Are OECD stock prices characterized by a random walk? evidence from sequential trend break and panel data models. *Applied Financial Economics* 15, pp. 547-556.

Narayan, P.K. and R. Smyth, 2007. Mean reversion versus random walk in G7 stock prices evidence from multiple trend break unit root tests. *Journal of International Financial Markets, Institutions and Money*, 17, pp. 152-166.

Newey, W. and K. West, 1994. Automatic Lag Selection in Covariance Matrix Estimation. *Review of Economic Studies*, 61, pp. 631-653.

Ozdemir, Z.A., 2008.Efficient market hypothesis: evidence from a small open-economy, *Applied Economics*, 40, pp. 633-641.

Perron, P., 1989. The great crash, the oil price shock and the unit root hypothesis. *Econometrica*, 57(6), pp. 1361-1401.

Shively, P.A., 2003. The nonlinear dynamics of stock prices. *The Quarterly Review of Economics and Finance*, 43, pp. 505-517.

Tong, H., 1983. Threshold models in non-linear time series analysis. *Lecture Notes in Statistics,* 21.

Tong, H., 1990. *Non-linear Time Series: A Dynamical System Approach*. Oxford University Press.

Urrutia, J.L., 1995. Test of random walk and market efficiency for Latin American emerging equity markets. *Journal of Financial Research*, 18, pp. 299-309.

**Table 1. Unit Root Tests without Trend and with Trend**

| Unit Root tests without trend | | | | | | |
|---|---|---|---|---|---|---|
| | Levels | | | First Difference | | |
| Method | ADF | PP | KPSS | ADF | PP | KPSS |
| P/D | -2.693(1) | -2.623(16) | 2.623[5]*** | -25.767(1)*** | -29.661(11)*** | 0.025[16] |
| Unit Root tests with trend | | | | | | |
| | Levels | | | First Difference | | |
| Method | ADF | PP | KPSS | ADF | PP | KPSS |
| P/D | -3.831(1) | -3.727(17) | 0.528[32] *** | -25.762(1)*** | -29.653(11)*** | 0.015[16] |

*Notes*: The number in parenthesis indicates the lag order selected based on the recursive t-statistic, as suggested by Perron (1989). The number in the brackets indicates the truncation for the Bartlett Kernel, as suggested by Newey and West (1994). *** denote the significance levels at 1%.

**Table 2. Wald Tests for a Threshold Effect in the Monthly US Stock Price–Dividend Ratio for Different Lags, m**

| M | $W$ | Bootstrap critical values | | | Bootstrap p-values |
|---|---|---|---|---|---|
| | | 10% | 5% | 1% | |
| 1 | 112.374 | 37.972 | 43.822 | 52.333 | 0.030 |
| 2 | 51.118 | 36.132 | 41.126 | 43.661 | 0.170 |
| 3 | 98.051 | 35.686 | 39.860 | 55.688 | 0.070 |
| 4 | 112.853 | 34.867 | 38.588 | 43.168 | 0.000 |
| 5 | 105.214 | 36.348 | 40.169 | 47.333 | 0.090 |
| 6 | 85.666 | 36.480 | 44.492 | 54.210 | 0.120 |
| 7 | 77.777 | 37.288 | 47.040 | 52.927 | 0.690 |
| 8 | 81.455 | 35.566 | 41.635 | 53.524 | 0.350 |
| 9 | 95.354 | 36.530 | 40.524 | 53.849 | 0.240 |
| 10 | 79.277 | 35.171 | 38.855 | 57.064 | 0.620 |
| 11 | 112.377 | 37.127 | 39.796 | 57.518 | 0.030 |
| 12 | 77.624 | 36.516 | 41.057 | 59.699 | 0.320 |

*Notes*: The second row gives the Walt-statistics, W, for different m. The third to fifth rows show the critical Wald-statistics at three significance levels according to a bootstrap approach with 5,000 replications. The last row presents the bootstrap p-values. The optimal delay is $\hat{m}$ = 4, highlighted in bold face.

**Table 3. Least-squares Estimates of Parameters of the Unconstrained Threshold Model with an Optimal Decay, $\hat{m}$ = 4**

| Regressors | Regime 1: $y_{t-1} - y_{t-4} \le -0.0717$ | | Regime 2: $y_{t-1} - y_{t-4} \ge -0.0717$ | |
|---|---|---|---|---|
| | Estimate | S.E. | Estimate | S.E. |
| 1 | 2 | 3 | 4 | 5 |
| $y_{t-1}$ | -0.029 | 0.012 | 0.007 | 0.003 |
| Intercept | 0.011 | 0.019 | -0.005 | 0.002 |
| $\Delta y_{t-1}$ | -0.075 | 0.092 | 0.365 | 0.027 |
| $\Delta y_{t-2}$ | -0.262 | 0.091 | -0.055 | 0.028 |

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| $\Delta y_{t-3}$ | -0.231 | 0.088 | 0.008 | 0.028 |
| $\Delta y_{t-4}$ | -0.475 | 0.099 | 0.102 | 0.028 |
| $\Delta y_{t-5}$ | 0.358 | 0.114 | 0.031 | 0.026 |
| $\Delta y_{t-6}$ | 0.262 | 0.111 | -0.019 | 0.026 |
| $\Delta y_{t-7}$ | 0.076 | 0.090 | 0.026 | 0.026 |
| $\Delta y_{t-8}$ | -0.120 | 0.104 | 0.038 | 0.026 |
| $\Delta y_{t-9}$ | 0.208 | 0.105 | -0.020 | 0.026 |
| $\Delta y_{t-10}$ | -0.033 | 0.083 | 0.024 | 0.027 |
| $\Delta y_{t-11}$ | -0.375 | 0.108 | 0.043 | 0.026 |
| $\Delta y_{t-12}$ | 0.093 | 0.091 | -0.074 | 0.025 |

Notes: The threshold estimate is $\hat{\lambda} = -0.0717$.

**Table 4. One and Two-sided Wald Tests for Threshold Unit Roots in the US Stock Price–Dividend Ratio for Different Lags, m**

| m | One-sided Wald test, $R_1$: Bootstrap critical values | | | | | Two-sided Wald test, $R_2$: Bootstrap critical values | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $W$ | 10% | 5% | 1% | *p*-values | $W$ | 10% | 5% | 1% | *p*-values |
| 1 | 13.67 | 8.66 | 10.89 | 13.73 | 0.040 | 13.67 | 7.79 | 10.16 | 13.73 | 0.040 |
| 2 | 12.55 | 8.98 | 10.42 | 17.62 | 0.040 | 12.14 | 8.12 | 9.61 | 17.62 | 0.040 |
| 3 | 5.82 | 9.47 | 11.29 | 16.85 | 0.240 | 5.82 | 7.70 | 10.07 | 16.85 | 0.290 |
| 4 | 10.10 | 8.65 | 9.68 | 12.18 | 0.030 | 10.10 | 8.14 | 8.58 | 11.67 | 0.050 |
| 5 | 7.26 | 7.90 | 10.27 | 13.69 | 0.110 | 7.26 | 7.67 | 8.76 | 10.91 | 0.130 |
| 6 | 5.58 | 8.27 | 9.92 | 11.84 | 0.290 | 5.58 | 7.21 | 9.82 | 11.84 | 0.350 |
| 7 | 7.19 | 9.11 | 10.40 | 11.60 | 0.240 | 6.51 | 8.12 | 8.69 | 11.60 | 0.260 |
| 8 | 6.06 | 8.67 | 9.51 | 12.38 | 0.320 | 5.74 | 7.96 | 8.82 | 10.53 | 0.320 |
| 9 | 5.06 | 9.15 | 11.46 | 12.31 | 0.310 | 5.06 | 8.04 | 10.52 | 11.88 | 0.330 |
| 10 | 5.14 | 10.22 | 11.19 | 13.76 | 0.350 | 4.99 | 10.20 | 11.19 | 13.76 | 0.370 |
| 11 | 5.35 | 8.72 | 11.50 | 13.51 | 0.300 | 5.35 | 8.35 | 10.66 | 12.55 | 0.360 |
| 12 | 6.26 | 9.77 | 10.46 | 13.72 | 0.260 | 5.82 | 8.42 | 9.75 | 10.70 | 0.300 |

*Notes*: The optimal delay is $\hat{m}$ = 4, highlighted in bold face.

**Table 5. Partial Unit Roots in the Monthly US Stock Price–Dividend Ratio for Different Lags, m**

| m | Bootstrap critical values | | | | | Bootstrap critical values | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $W$ | 10% | 5% | 1% | *p*-values | $W$ | 10% | 5% | 1% | *p*-values |
| 1 | 1.81 | 2.27 | 2.62 | 3.59 | 0.240 | 2.22 | 2.33 | 2.54 | 2.99 | 0.180 |
| 2 | -0.64 | 2.41 | 2.77 | 3.09 | 0.880 | 3.48 | 2.49 | 2.66 | 3.62 | 0.020 |
| 3 | 1.21 | 2.04 | 2.35 | 2.74 | 0.380 | 2.09 | 2.53 | 2.83 | 4.03 | 0.230 |
| 4 | 2.37 | 2.31 | 2.53 | 2.93 | 0.100 | 2.12 | 2.45 | 2.70 | 2.88 | 0.190 |
| 5 | 1.58 | 2.43 | 2.69 | 3.27 | 0.250 | 2.18 | 2.25 | 2.52 | 2.96 | 0.140 |

| m | Bootstrap critical values | | | | | Bootstrap critical values | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | *W* | 10% | 5% | 1% | *p*-values | *W* | 10% | 5% | 1% | *p*-values |
| 6 | 0.24 | 2.24 | 2.48 | 2.86 | 0.640 | 2.35 | 2.41 | 2.80 | 3.29 | 0.140 |
| 7 | -0.83 | 2.36 | 2.66 | 2.86 | 0.850 | 2.55 | 2.67 | 2.76 | 3.28 | 0.140 |
| 8 | -0.57 | 2.62 | 2.75 | 2.97 | 0.800 | 2.40 | 2.47 | 2.62 | 2.70 | 0.110 |
| 9 | 0.27 | 2.43 | 2.66 | 2.97 | 0.670 | 2.23 | 2.33 | 2.68 | 3.27 | 0.120 |
| 10 | -0.39 | 2.40 | 2.70 | 3.52 | 0.800 | 2.23 | 2.39 | 2.77 | 3.37 | 0.160 |
| 11 | 1.17 | 2.35 | 2.59 | 3.26 | 0.460 | 1.99 | 2.33 | 2.80 | 3.35 | 0.250 |
| 12 | -0.67 | 2.31 | 2.68 | 3.03 | 0.850 | 2.41 | 2.45 | 2.89 | 3.19 | 0.110 |

Notes: The optimal delay is $\hat{m}$ = 4, highlighted in bold face.

**Figure 1. Stock Price-Dividend Ratio Monthly Data for the US Stock Market (1871:M01 to 2012:M03)**

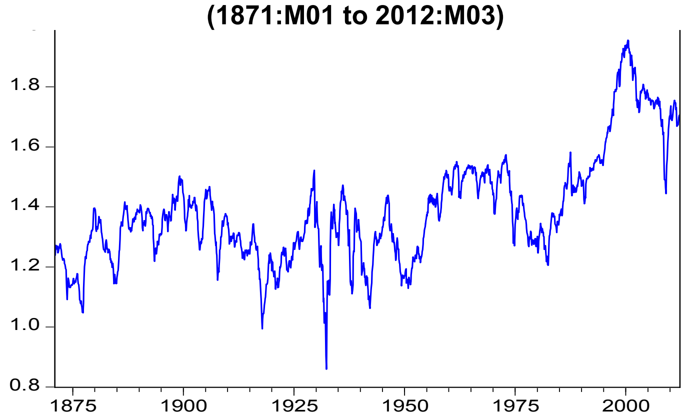


**Figure 2. Monthly Data for the US Stock Market, 1871:01 to 2012:03, Classified by Threshold Regime**