I. CONTROVERSIES OVER THE SIZE OF THE PUBLIC BUDGET[1]

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Abstract

This paper examines two potential positions in the debate over the public budget size. Notably, the indicator used is inherently unstable, and shifting opinions regarding the public budget's size might be a psychosocial mirror of the real-world volatility itself. However, this indicator may be dominated by a hidden variable in given historical frameworks, and the ensuing debates under this hypothesis might reflect the effective public budget's deviations from the applicable contextual attractor. Testing the temporal features of a large database, the paper supports the second assumption. The possible attractors are approximated using the VAR algorithm and the BARS Curve.

Keywords: public budget size, attractor, VAR, BARS curve

JEL Classification: C32, E62, H60

1. Introduction

Public finances have become a major issue in economic debates. The controversies concern all the relevant components, such as the main uses for government revenues (*e.g.*, personal security, protection of property, education, health care, social welfare protection, national defense, and infrastructure), the forms and the taxpayer distribution of the fiscal burden, and the decisional hierarchy.

Of particular interest is the influence of budget expenditures on global output.

Three evaluations predominate in this regard.

- Many authors (Beach 1998; Lynch 2004; Brooks and Hwong 2006; Myles 2009; Keen 2012; Pettinger 2012) insist on the direct or contingent beneficial effects of public expenditures on economies, either as demand-side incentives (based on the Keynesian paradigm) or as supply-side factors (such as accumulation of human capital, research programs, and development of infrastructure).

- The negative consequences of public expenditures are also frequently invoked; see Mark *et al*. (1997), Beach (1998), Pettinger (2012) and McBride (2012). Poulson and Kaplan (2008) have cited many studies that credit this perspective.

- Opinions attesting to the ineffective (at best) roles of budget expenditure and tax administration structures in economic growth also cannot be ignored; see Engen and Skinner (1996), Mark *et al*. (1997), Wasylenko (1997), Xing (2011) and ITEP-Institute on Taxation and Economic Policy (2011). The Harberger theorem can be categorized in the same mainstream; see Harberger (1964, 2003), Mendoza *et al*. (1997) and Hines (2002).

This diversity emerges in a long series of case studies (Appendix 1); for similar syntheses, see Husnain *et al*. (2011) and McBride (2012). In addition, debates concerning the size of the public budget cannot be considered separately from the effective state of the particular economies involved in these debates.

The main empirical material for economic investigations is provided by a huge network of national and international statistics that numerically describe the evolution of different countries, different geo-political zones, and the world as a whole. In other words, reality itself offers cognitive support (impulses and falsifiable information) for new economic statements.

On the other hand, the propensity of economic research to move towards current, broad-based topics of interest is commonplace and understandable and reflects, inter alia, public opinion. Its influence on politics – consistent and visible under democratic systems, weaker and (more or less) hidden in authoritarian regimes – has been extensively outlined by several sociological and communicational studies; see Wu and Huberman (2006, 2008), Rashott (2007), Arnold (2007), Hagen (2008), Manza and Brooks (2012), Johnston and Ballard (2014) and Davison (2015). Thus, government revenues and expenditures represent one of the most important instruments of macroeconomic management and are, in turn, intimately connected to political institutions; see Birney *et al*. (2008), Wlezien and Soroka (2009), Stachowiak (2013), Moy and Bosch (2013), Grant (2014) and Bølstad (2015).

If the diversity of opinions about budget size↔global output interactions is deeply linked to the real state of the economy, it is natural to search for its inducing factors in the characteristics of the state itself. Approaching this topic in a simpler manner, two hypotheses seem promising.

First, the possibility that establishing the size of the public budget is an intrinsically irregular phenomenon cannot be excluded. Under this type of assumption, the presence of shifting and often contradictory opinions regarding this issue, as such, might be interpreted as a psychosocial mirror of real-life volatility.

However, in terms of the second hypothesis, many theoretical and empirical studies have revealed objective determinants (obviously acknowledged in the social sciences) of the size of the public budget. Under a given historical context, it is thus possible that the volatility of this indicator is dominated by a sui generis hidden attractor. Under such a hypothesis, the controversies surrounding the size of the public budget might reflect, among other factors, the position of the effectively recorded size against the respective contextual attractor.

The crucial query of this paper thus becomes to check whether the time series regarding the ratio of public expenditures to GDP (cbe) are characterized by stable features.

What might define this representativeness on a statistically relevant sample? Two criteria have been considered:

- First, the data must be a numerical image of a "given historical context". This context is interpreted as a temporal interval of a generation length and is characterized by similar demographic, technological, socio-economic, cultural, geo-strategic, and environmental features and by other factors that significantly influence the variable of interest. Consequently, the period between 1990 and 2012 is adopted as the empirical sample period.

- Second, the sample must cover a span of circumstances as wide as possible under which the variable of interest was recorded (*e.g.*, demographic potential, natural resources, degree of economic development, geopolitical status, and local peculiarities). To this end, statistical series for 95 countries on all continents (Appendix 2) were obtained. Since the World Bank thesaurus (World Development Indicators, 2015) is characterized by many enough lacunae, especially regarding cbe, the data of International Food Policy Research Institute (IFPRI 2015) were also used. This information was included as such or slightly corrected (taking into account the relative differences registered between IFPRI and WB statistics in segments of common information). For Romania, we used the estimations of cbe published in Dobrescu (2015a). Attempts to acquire a more extensive database failed because of informational limitations.

To identify not only the properties of the public budget size series themselves but also their possible functional links to other macroeconomic variables, four indicators have been employed:

- Three indicators are directly involved in various econometric analyses: the aforementioned ratio of public budget expenditures to GDP (cbe); the index, in real terms, of the gross domestic product (IGDPc); and the index, again in real terms, of gross capital formation (IGCFc), estimated under the assumption of equality between the GDP and GCF deflators.

- A fourth indicator, per capita GDP in current US$, is used as auxiliary information

Because some statistical series were incomplete, an ad hoc procedure of forward and backward extrapolations was applied to approximate the unavailable information (the respective methodology is described in Appendix 3; the integral database is detailed in Supplementary material S1).

At first glance, the database confirms the volatility of the cbe time series. The series' coefficient of variation was below 5% in only 7.3% of cases; it was between 5% and 10% in 35.4% of cases; and the remaining cases (57.3%) exceeded the 10% limit. Therefore, the data do not reject ab ovo the supposition that the size of the public budget might represent an intrinsic irregular phenomenon. The coefficient of variation, however, does not provide sufficient reasons to accept this statement as a well-founded conclusion. The second chapter extends the analysis and involves more refined techniques to reveal the temporal features of the examined series.

Among these techniques, the testing procedures for stationarity, random walk and serial correlation are prioritized. This choice is not fortuitous; a stationary series is suitable for a large range of linear econometric specifications and computational algorithms. In contrast, a random walk series is, by definition, unpredictable under such procedures. In the case of serially correlated series (regardless of whether they are stationary), either the lengthy autoregressive processes or non-linear modeling might be more successful as approaches.

The random walk hypothesis was initially examined with the variance ratio test. Three options were thus processed: "random walk" for the primary series, "exponential random walk", and "random walk innovations".

The acquired information was correlated with the results of stationarity procedures, including the Augmented Dickey-Fuller (ADF), Phillips-Perron (PP), Kwiatkowski-Phillips-Schmidt-Shin (KPSS), Elliott-Rothenberg-Stock (ERS), and Ng-Perron (NP) tests. Regarding exogeneity, these statistics are determined using common variants: none, constant, and constant plus trend.

The Ljung-Box Q statistics extend the informational imaging of the temporal properties of the cbe series.

Special consideration is given to the Brock, Dechert, Scheinkman and LeBaron (BDS) test as a powerful tool for detecting serial correlation in the data. This procedure is applied under three options ("fraction of pairs", "standard deviations", and "fraction of range") with five dimensions (2, …, 6) and for both normal and bootstrap probabilities.

The H (Hurst exponent) is also calculated using an ad hoc methodology for increasing the number of rescaled range estimations. The ARFIMA algorithm has also proven to be a useful tool in this analysis.

The second chapter ends with the conclusion that for most of the examined series, the probability to be dominated by more or less stable and predictable patterns is significant.

Consequently, the third chapter attempts to find possible attractors in the cbe statistical series, focusing investigations on two computational techniques.

One technique revolves around the VAR algorithm, estimating the steady state of the given series by an asymptotical value of a moving average auto-regressive process. This type of attractor will be symbolized as ATT1. Under this procedure, it initially determines the set of stable VARs (if any are available), the roots of the resulting characteristic polynomial lying inside the unit circle. The coefficients of the longest VAR from this set are used for successive extrapolations to establish the long-term pattern of the series, including its steady state (again, if any exists).

The second approach can be considered structural, as it is inspired by the so-called BARS Curve, which assumes an intrinsic interdependence between public budget expenditures and the global output of the economy. The optimal point of such a curve is considered a possible attractor, and this type of attractor will be noted by ATT2.

The final chapter connects the above-described analyses to the primary subject of the paper – the epistemological sources of the controversies surrounding the size of the public budget.

In a database covering 95 countries and almost an entire quarter of a century, the paper identifies signs of random walk behavior in the examined series. These signs, per se,

might suggest that the above-mentioned controversies do not have economic roots as such and are rather an expression of the volatile psychosocial and political environment. Nevertheless, the dominant finding is a serial correlation in most of the data, which substantially credits the presence of possible attractors as gravitational points of the variable in question. Under such circumstances, the oscillations of social perceptions (including academic debates) concerning the size of the public budget might be more consistently explained by the effective comparative position of a given economy with the respective attractors. Their estimations should also have an important operational significance, particularly in building public finance policies.

2. Temporal Properties of the cbe Series

The principal goal was to reveal statistically the nature of the ratio of public expenditures to GDP (cbe). Is this time series inherently irregular? Alternatively, is it engrafted on a certain pattern (that is more or less visible)?

It is essential to identify the temporal behavior of the cbe series as such. In other words, the data in levels have priority, and the differences (of any order) are considered of secondary importance.

These series are initially checked for the random walk hypothesis with the variance ratio test. The results obtained are also corroborated with the stationarity procedures. The autocorrelation, Ljung-Box Q statistics, BDS test, coefficient d from the ARFIMA algorithm, and the Hurst exponent are used supplementary.

2.1. Variance Ratio Test

The most commonly used procedure to test a statistical series for random walk is the variance ratio. Based on the model $y_t=c+y_{t-1}+e_t$ (in which c is a drift constant and e_t are uncorrelated innovations with zero mean), this test assumes as the null hypothesis that a univariate time series y is a random walk. This test "exploits the fact that the variance of the increments of a random walk are linear in the sampling interval" (Lo and MacKinlay 1988, p. 43) and is occasionally appreciated as "more powerful than unit root tests" (Lam *et al.* 2006, p. 2). Notable applications or extensions of this test can be found in Seungmo (1998), Cerrito *et al.* (1998), Busetti and Harvey (1998), Perron (1988), Arlt and Arltova (2000), Kim (2004), Tas and Dursunoglu (2005), Nakamura and Small (2007), Chen (2008), Kavalerchik (2010), Charles and Darné (2009), Oskooe (2011) and Li and Liu (2012).

The null hypothesis probability has been computed for the statistical series as such (the series is a martingale) and for their logarithms (the log of the series is a martingale). In the case of the primary data, the corresponding symbols are VRTS and VRTL, whereas for those computed using wild bootstrap, the corresponding symbols are VRTSb and VRTLb. The results are presented in Figure 1.

The VRT Distribution of the Probability of the Null Hypothesis
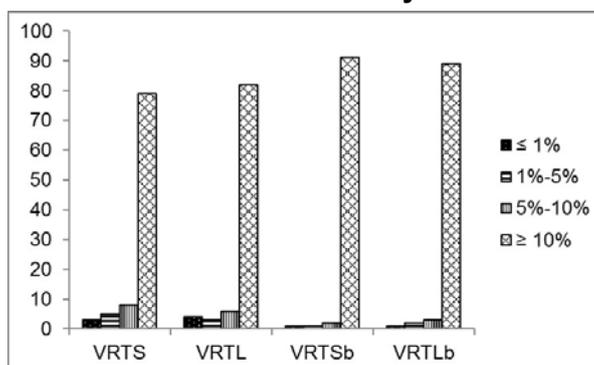
The probability of the null hypothesis (series as such and the log are martingales) is thus very high in both computational variants – primary or bootstrapped data in most cases. The presumption of a possible unpredictable evolution of the public budget size thus obtains non-negligible statistical support. Theoretically, a random walk series is not stationary; therefore, the data examined here will also be submitted to unit root tests.

2.2. Unit Root Tests

A battery of five tests was used to clarify whether the cbe series are stationary in level. We began with the algorithm developed by Dickey and Fuller (see Dickey and Fuller 1979, 1981; Said and Dickey 1984), the ADF test. The statistics for all series have been determined in three variants of exogenous hypotheses: none (suffix n), constant (suffix c), and constant plus trend (suffix ct). Supplementary material S2 presents the results obtained.

The results show that the probabilities of the null hypothesis (series in level has a unit root) are grouped as follows: ≤1%, 1%–5%; 5%–10%, and ≥ 10%. The last group, therefore, appears to be the most populated.

Another similar procedure that is more appropriate for short series is the PP test (Phillips and Perron 1988). This test refers to the same null hypothesis as the ADF test. The results (Supplementary material S2) are analogous to those of the ADF test.

The KPSS technique (Kwiatkowski *et al*. 1992; Cappuccio and Lubian 2009) attempts to mitigate the weaknesses of Dickey-Fuller type tests regarding the stable autoregressive (with roots near unity) or fractionally integrated alternatives (Kwiatkowski *et al*. 1992, p. 160). The KPSS technique is typically computed for two exogenous tests: constant (KPSS) and constant plus linear trend (KPSSct).

According to its creators, KPSS is "intended to complement unit root tests, such as the Dickey-Fuller tests" (Kwiatkowski *et al*. 1992, p. 178). Similar considerations can be found in Cheung *et al*. (1994).

The KPSS estimations in our sample (Supplementary material S2) display an important share of cases with a probability of null hypothesis that is >10%. The results of Dickey-

Fuller type and KPSS tests cannot be directly evaluated because they are centered on different null hypotheses.

The results of the ERS test (Elliott *et al.*, 1996; Elliot and Jansson, 2003), which is also presented in Supplementary material S2, again reveal the clear predominance of the group of series suspected for the presence of unit root.

The last econometric technique for checking stationarity is the NP test (Ng and Perron, 1995). Its goal was to avoid problems generated by the previous testing techniques, particularly the following:

a) Low power when the root of the autoregressive polynomial is close to but less than unity, and

b) Severe size distortions when the moving average polynomial of the first differenced series has a large negative root (Ng and Perron, 2001, p. 1520).

The NP test, which is a prolongation of the ERS procedure, is "based upon the local GLS de-trending method but also uses an autoregressive spectral density estimator of the long-run variance"; see Crowder (2001, p. 4).

Computed for constant and also for constant and linear trend, this procedure also supports the non-stationarity presumption.

The public budget size in relative terms belongs to the class of bounded time series $(0 > cbe > 1)$. Under such conditions, researchers have noted that the standard unit root tests "might become useless if time series are close to the boundaries" (Carrion-i-Silvestre and Gadea, 2010, p. 2). This issues does not emerge in our case because the series evolves significantly far from zero and unity, particularly given that the involved exogenous values are "none" or "constant". The results for linear trend are also presented.

An overall picture of our application is shown in Table 1, which contains the distributions of the main intervals of the null hypothesis probability for all types of unit root tests.

Table 1

Shares of the Main Intervals of the Null Hypothesis Probability

Test	Intervals				
	≤ 1%	1%–5%	5%–10%	≥ 10%	Sum
ADF	0.066667	0.094737	0.045614	0.792982	1
PP	0.035088	0.091228	0.052632	0.821053	1
ERS	0.1	0.078947	0.031579	0.789474	1
NP	0.098684	0.096053	0.122368	0.682895	1
KPSS	0.005263	0.405263	0.121053	0.468421	1

According to the ADF and PP tests, an overwhelming proportion of the examined time series cannot be considered stationary. Despite several slight differences, a similar conclusion is also validated by the ERS, NP, and KPSS tests. This conclusion is congruent with the previously discussed results provided by the variance ratio test because the random walk series are inherently non-stationary. As with other empirical studies, we nevertheless remain cautious, particularly concerning the possible presence of long-term interdependencies in the data, regardless of whether these interdependencies are linear with high-order lags.

## 2.3. Ljung-Box Q Statistics and the BDS Test

The Ljung-Box Q statistic is a useful tool for testing autocorrelation with multiple lags; see Ljung and Box (1978), Andrei (2003), Kan and Wang (2008), Hyndman (2014) and MathWorks (2016). Its null hypothesis is that "there is no autocorrelation up to given lag order".

This test has been computed for all countries using 12 lags – a number sufficiently large to cover a wide band of possible temporal interdependencies (note that yearly data are employed). The distribution of the estimations obtained in this manner (95x12) is presented in Supplementary material S3. Overall, the probability of the null hypothesis is greater than 10% in 21-22% of the cases. In more than 73% of the cases, it does not exceed 5%, which implies an extensive presence of serial correlation in the data.

Another powerful technique to reveal interdependencies in the data is the BDS test; see Brock *et al*. (1996), Sprott (2003) and Kuan (2008). Considering problems highlighted in the literature regarding this technique (Belaire-Franch and Contreras, 2002; Caporale *et al*., 2004; de Graaff *et al*., 2006), the BDS statistic was estimated for all cbe series using three computational methods ("fraction of pairs", "standard deviations", and "fraction of range"), and five embedding dimensions (2, 3, 4, 5, and 6). Only the distance ε has been maintained constant at 0.7 (a default value in the Econometric Views statistical package). This procedure has been performed for both normal and bootstrap probabilities.

Figure 2

Distribution of the Null Hypothesis Probability
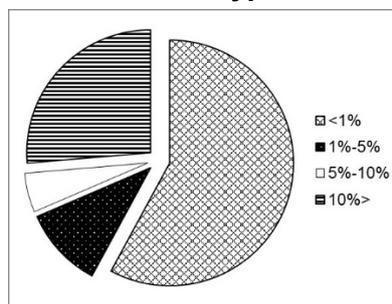
The results are provided in Supplementary material S4. Here, we focus upon the distribution of the null hypothesis probabilities – the data are independent and identically distributed (i.i.d.) – aggregated for the following thresholds: <1%, 1%–5%, 5%–10%, and greater than 10%. Each interval is presented as a share of the total processed cases.

Globally, the distribution for all cases (95x3x5x2=2850) is reproduced in Figure 2.

The null hypothesis is rejected in most cases; the share of the thresholds below 1% and between 1% and 5% equals 0.685. From this perspective, the most trenchant proved to be the "standard deviations" method (0.822).

Regarding the primary data as such, the share of the first two groups of the null hypothesis probability exceeds 0.81. Notably, we should recall that the bootstrap

procedure attenuates this proportion significantly (0.56) in favor of higher probabilities of the null hypothesis.

2.4. Hurst Exponent and Parameter 'd' in the ARFIMA Model

As a supplementary check of the temporal characteristics of the studied dataset, we have also estimated the Hurst exponent (H), while remaining conscious of the limits of such an approach under the relative brevity of the available statistical series.

This parameter takes values between zero and unity, and its variation is typically understood as follows: "H=0.5 corresponds to a random-walk process which means no memory in the time series. 0<H<0.5 indicates large and small values of the time series are more likely to alternate. If 0.5<H<1, there are persistent long-range power-law correlations in the time series" (Duan and Stanley, 2010, p. 4). Similar or slightly nuanced commentaries about the interpretation of the H can be found in Sánchez Graneroa *et al*. (2008); Racine (2011); Dasmane and Valeinis (2011); Barunik and Kristoufek (2012); Mansukhani (2012); Kaplan (2013) and Mynhardt *et al*. (2014).

Generally, the cognitive virtues of the H are strictly concerned with the features of the examined series, with no predictive valence (Kaplan, 2013).

There are many techniques to approximate the H: rescaled range analysis, de-trended fluctuation analysis, wavelet-based estimation, the geometrical method, the de-trending moving average, and period-gram regression (Sánchez Graneroa *et al*., 2008; Dasmane and Valeinis, 2011). We applied the widely used R/S procedure; see Taqqu *et al*. (1995), Qian and Rasheed (2004), Lenskiy and Seol (2012), Barunik and Kristoufek (2012), Mansukhani (2012), Kaplan (2013) and Mynhardt *et al*. (2014). Eight terms are considered the minimum length of the smallest time series for which R/S is computed (Mansukhani, 2012). In our application, the standard R/S methodology (Pipiras and Taqqu, 2016, pp. 86-89) is processed with adjustments (Appendix 4 and Supplementary material S5).

Figure 3 presents the distribution of the Hurst exponent in ascending order (AEH), aggregated into four representative intervals: <0.5; 0.5-0.65; 0.65-0.8; and 0.8-1.

Figure 3

Distribution of the Hurst Exponent
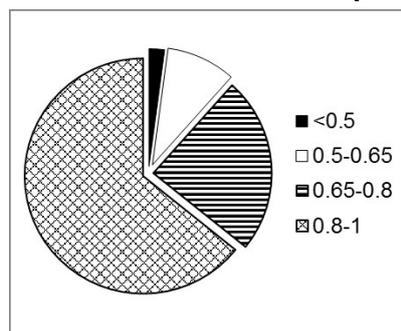
Thus, the share of H values less than 0.5 is practically insignificant, and the share of those corresponding to a random-walk process (close to 0.5) is relatively small. In

contrast, the cases characterized by persistent long-range power-law correlations (0.5<H<1) are preponderant. Nevertheless, we must be cautious in interpreting these results not only because of the shortness of cbe statistical series but also because of the problems with the algorithm itself. It was shown that when using R/S analysis, "it is possible to obtain evidence of long memory in random series"; see Sánchez Graneroa *et al*. (2008, p. 5544).

In recent decades**,** attempts to identify a serial correlation in economic (particularly financial) time series using the family of autoregressive fractionally integrated moving average (ARFIMA) models has intensified; see Galbraith and Zinde-Walsh (2001), Wagenmakers *et al*. (2005), Torre *et al*. (2007), Assa *et al*. (2015) and Graves *et al*. (2015).

Because the present study aims primarily to identify the serial correlation in cbe data and not their best adjustment by regression, a simple econometric specification was adopted, including only the intercept and parameter d. In several cases, dummy variables were included to attenuate the effects of outliers. The values of the d parameter for all countries are presented in Supplementary material S6. Their distribution is provided in Figure 4.

<div align="right">**Figure 4**</div>

Distribution of the d Parameter
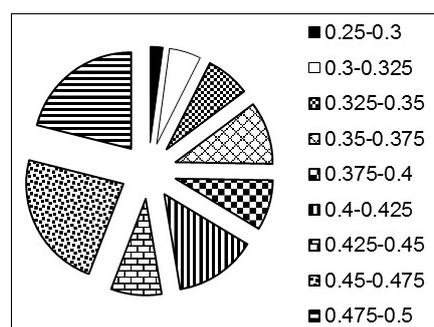
In all cases, parameter d is between 0.25 and 0.5, which undoubtedly attests to the presence of long serial correlation in the data. This property was identified early in the 1980s by Granger-Joyeux (1980) and Hosking ("When 0<d<1/2, the ARFIMA process is a stationary process with long memory", Hosking, 1981, p. 169). For similar considerations, see also Andersson (1998, p. 5), Panas (2001, p. 398), Baum (2013, p. 23) and Nazarian *et al*. (2014, p. 18-19).

Two main conclusions can be derived from this chapter. The assumption that the relative public budget size would follow a random walk evolution must be categorically rejected. Most statistical tests have signaled serial correlation in the corresponding data. Moreover, such a temporal property instead appears to be valid in the long term, although in some cases, the short-to-medium term is also plausible.

Under these conditions, we cannot ignore the second possible path to be explored, namely, the possibility of an attractor's presence in the corresponding time series.

3. Searching for Possible Attractors

As we have previously remarked, the debates surrounding the size of the public budget could be linked with the deviations of its actual dimensions caused by a non-observable attractor. This possibility will be examined here in two approachable hypotheses:

● As a steady-state level in an autoregressive process

● As an extreme point of a structural relationship between cbe series and other significant macroeconomic variables.

3.1. Autoregressive Process

This methodology revolves around the univariate autoregressive technique; see Dobrescu (2015b) for references.

Its initial step consists of identifying the set of stable VAR, the longest of which is chosen (LSVAR). Both features – stability and the maximal length – are compulsory. The first allows the convergence of post-sample extrapolations, whereas the second is important for the representativeness of the econometric estimations. Briefly, the computations must be based on a VAR with the most lags and for which all the roots of the corresponding characteristic polynomial lie inside the unit circle.

Three series (GRCcbe, UKRcbe, and ZAFcbe) did not observe the stability condition. The remainder (92 cases) did; the number of lags characterizing the longest of these is presented in Appendix 5. Almost one-fourth of LSVARs have the maximum possible number of lags (10), and more than one-fourth have 8-9 lags; less than a decile represents the lowest category, with 2-3 lags. In other words, the representativeness of identified LSVARs is sufficient.

The second step of the methodology consists of successive forecasting estimations based on LSVAR coefficients (Supplementary material S7). The estimations derived from the LSVAR formula in prolongation of the respective time series will be named, "post-sample extrapolations".

On the one hand, such estimations are useful for approximating the searched steady state; on the other hand, they are also useful for decrypting the long-term pattern of the given time series. In autoregressive interpretation, the attractor is assimilated to the steady-state level, which is computed by post-sample extrapolations. Thus, the following question arises: How can such a level be defined?

An easy (but not trivial) solution would be to extend the post-sample extrapolations until a relatively stable result is obtained with a sufficient degree of confidence. In our application, it is assumed that such a milestone is reached when at least one hundred consecutive post-sample extrapolations have d1 (the first-order difference in module) less than or equal to 1e-4. A higher precision appears to be unhelpful from a practical perspective.

Applied to the cbe series, the presented methodology shows that only four cases do not observe the established coordinates for approximating the steady state. Three of these – GRCcbe, UKRcbe, and ZAFcbe – have already been mentioned; the corresponding series did not satisfy the stability condition. Another (CYPcbe) tends to stabilize at a negative value, which flagrantly contradicts the admissible socio-economic boundaries of this variable.

With regard to such cases, we do not suggest that an attractor would be generally excluded. Our statement refers only to the autoregressive technique, which proved unable to identify the attractor. Therefore, this group of series will be named "*AR unstable or generating negative steady-state*".

The remainder of the sample (91 countries) complies with the VAR stability condition and provides steady-state values ranging between zero and unity.

As expected, the number of post-sample extrapolations necessary to reach a steady-state level varies in a very large range – from 102 (BLZcbe) to 2519 (LUXcbe). Supplementary material S8 details this information, revealing a high concentration of PSE values in the lower classes of a one-tabulation distribution.

The level of PSE might be linked with many potentially explicative variables, of which we shall examine three:

  – Length of the stable VAR

  – Mean sample

  – Precision degree imposed exogenously to identify the steady-state level

For a more formal examination, the following indicators will be involved:

MS – mean sample;

PSE – post-sample extrapolation, computed obviously for the same time frequency as the sample

m – number of LSVAR lags

PD – precision degree (exogenous), established as a maximal admissible first-order difference between two successive post-sample extrapolations

PDP – precision degree parameter

A possible inter-country comparability might be obtained, for instance, by calculating PDP as a relative measure:

$$PDP=PD/(PD+MS) \tag{1}$$

Therefore, PDP falls between 0 for PD=0 and 1 for PD$\to \infty$ [from PDP=1/(1+MS/PD)], which appears acceptable.

Supplementary material S9 displays the cross distribution of PSE and each of the mentioned factors (m, MS, and PDP).

Short comments:

● If the data are slightly compressed –, respectively (m=2, 3, and 4), (m=5, 6, and 7), and (m=8, 9, and 10) – the proportion of the first group in the total increases from 13.19% to 35.16% in the second and to 51.65% in the last. Within each group, the share of cases in which PSE<500 is overwhelming. In other words, the longer that the VAR is stable, the shorter the number of necessary post-sample extrapolations appears to be.

● More than four-fifths of the cases, with mean sample within 0.1-0.4, generate 100-500 post-sample extrapolations. The largest part of the other (two-thirds) belongs to the same class.

● Supplementary material S9 shows clearly that for a decreasing precision degree parameter (higher PDP), PSE does not rise (as perhaps was expected) but in fact decreases. Almost 84% of cases are concentrated within 2e-05<PDP<6e-05.

Whether such correlations represent only a particular statistical curiosity or have deeper significance remains a theme for further analysis.

An econometric estimation can also be useful. In a simple linear regression, PSE as the explained variable is related to m, MS, and PDP as explanatory variables. For eight countries (AUT, BGR, BHS, BTN, IRL, LKA, LTU, and MAR), there were attached dummies, considering their likely outlier status (Appendix 6). With some indulgence (for m), the statistical significance tests appear acceptable. The standardized coefficients reveal a notable contribution in determining PSE for all explanatory variables. The variance inflation factors also do not raise any particular collinearity problems.

Returning to the main issue, the attractors of the public budget size – in the above VAR interpretation – are presented in Appendix 7 for all 91 retained countries (after eliminating the cases with unstable VAR that induce a negative steady state).

Because it is determined using statistical data, it would be logical to examine the steady state in relation to this informational source. Some authors only outline the sample mean as a satisfactory approximation of the steady-state value.

There are two such possible anchors: the usual sample mean (MS) and the mean of the last m observations, corresponding to stable VAR lags (MLSLS). Both are implied in the post-sample extrapolations, the first through the VAR coefficients and the second as an actual starting point of computations. In our application, the differences between them do not appear to be essential (Figure 5).

Figure 5

The Sample Mean (MS) and the Mean of the Last m Observations Corresponding to Stable VAR Lags (MLSLS)
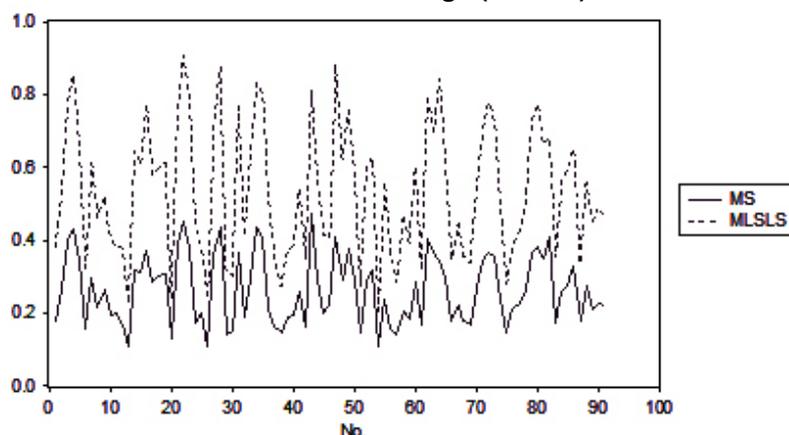
As a consequence, the focus was on the relationship between the estimated attractor and the sample mean.

With this aim, a relative magnitude, attractor-mean deviation (AMD), will be introduced:

$$ATT1 = \sum ESP_j / q \quad (j = k+1, k+2, \ldots, k+q) \tag{2}$$

where:

ATT1 – attractor in VAR acceptation (compulsory positive value)

q – precision set (exogenous), determined by the number of successive post-sample extrapolations that meet the requirement $d1(PSE_i)<PD$ in module

k – ante precision set, defined as the number of post-sample extrapolations until the beginning of the precision set

AMD – attractor-mean deviation

$$AMD=rAM*(1+f(PDP)+f(ESP)] \tag{3}$$

rAM1 – relative attractor-mean deviation for the attractor in VAR interpretation

$$rAM1=(ATT1/MS-1) \text{ in module} \tag{4}.$$

The values of rAM1 for all 91 cbe series admitting stable VAR are provided in Appendix 8. There are 19 cases characterized by a rAM1 under 0.01, 35 by $0.01<rAM<0.05$, 10 by $0.05<rAM<0.1$, 207 by $0.1<rAM<0.75$, and 7 by a higher value.

The last threshold was not chosen accidentally. The maximum registered statistical coefficient of variation does not exceed 0.8 (i.e., it equals exactly 0.78888). In other words, although we might admit such a limit as a norm for rAM1, we must recognize that the cases with higher levels could be suspected of socio-economic implausibility. As a result, in addition to the above-mentioned groups, it would be sensible to delimit a new group characterized by a *convergence towards unlikely steady-state (CUSS)* and consisting of the cases that generate VAR estimations with a relative attractor-mean deviation of over 0.75. Undoubtedly, such a concrete ceiling must be contextually established.

In such a situation, we have identified 7 cbe series, namely (the rAM is specified in brackets) IRNcbe (2.225356), ISLcbe (0.824267), JORcbe (3.448416), LSOcbe (3.512598), MDVcbe (2.290595), MNGcbe (1.3510), and NZLcbe (0.954531).

The examination of the rAM1 problem will be concluded by a short econometric incursion. What could be its main determinants? From a multitude of potential candidates, we select two for analysis: the sample coefficient of variation (CVS) and a new proposed indicator, the extrapolating sluggishness parameter (ESP). The latter is approximated as follows:

$$ESP=k/(k+n) \tag{5}$$

where:

k – ante precision set, defined as the number of post-sample extrapolations until the beginning of the precision set

n – sample dimension (number of statistical observations)

ESP varies between 0 for k=0, and 1 for $k\rightarrow \infty$ [from $ESP=1/(1+n/k)$]. To mitigate the outlier effects, a simple two-factor linear specification (rAM1=f(CVS, ESP) will be completed, with dummies for IRLcbe, IRNcbe, ITAcbe, LCAcbe, LKAcbe, LSOcbe, MDGcbe, MLIcbe, NORcbe, and SWEcbe (Appendix 9).

The results confirm the sample standard deviation and the extrapolating sluggishness parameter (per the above-stated definition) as significant explanatory variables of rAM1.

Another question worth noticing is as follows: "Do the post-sample extrapolations register a monotone or oscillatory trajectory towards the steady-state level?"

Strictly speaking, a series is considered to be evolving monotonically if the algebraic signs of the first-order differences do not change (when, independent of t, there are

observed inequalities $y_{t+1} \geq y_t$ for an increasing trend or, conversely, $y_t \geq y_{t+1}$ for a decreasing trend). Relaxing this interpretation somewhat, in our application, the series in which the mentioned algebraic sign alternates are also admitted as monotone but only for few initial terms of extrapolation (when the sample volatility effect remains significant).

Nine cbe series were retained with a "monotone convergence towards steady-state":

- LTUcbe and PRTcbe with the asymptotically increasing trajectory towards steady state
- AUTcbe, ISRcbe, LKAcbe, MYScbe, NAMcbe, SYCcbe, and VCTcbe with an asymptotically decreasing trajectory

The oscillatory trajectory is characterized by a repeated alternation of the algebraic sign of the first-order differences (d1). As a result, the post-sample extrapolations constitute a cyclical dynamic.

To outline more clearly this evolution, we delimited the so-called peak-values among the post-sample extrapolations. In the acceptation adopted here, a peak value is admitted for every post-sample extrapolation in which d1>0, which is *immediately* followed by another with negative d1.

The set of post-sample extrapolations located between two peak values forms a primary cycle (EC). Its length is defined by the number of terms included in such a set. Two or more consecutive primary cycles can be aggregated into a multiple cycle (CEC – couple, triplet and so on). Naturally, for a given series, all such multiple cycles must have the same dimension (two, three or other constant number of primary cycles).

The primary cycles – or their multiples – can contain a relatively uniform (obviously in a statistical sense) or, in contrast, a differentiated number of terms. The coefficient of variation for these lengths is subsequently determined for the primary cycles as such (CVEC) or for their multiples (CVCEC).

This coefficient of variation can be used as a discerning criterion between regular and irregular oscillation. For the moment, such a separating threshold will be exogenously settled. As a purely working convention, we will adopt 0.1 for this role. In other words, series with CVEC or CVCEC less than 0.1 are considered regular oscillation and, symmetrically, those with a variation coefficient higher than 0.1 are considered irregular.

Appendix 10 presents the list of cbe series classifiable as "*regular oscillatory convergence towards steady-state*", along with the necessary supporting information. This group contains 51 cbe series. The coefficients of variation indicate, therefore, that in the case of 29 cbe series, the superior border (ad hoc adopted as 0.1) was reached at the primary cycle level (marked bold italic). For the remainder (22 countries), multiple cycles (compounded from 2-3 primary cycles) were computed.

The alternative dynamics – "*irregular oscillatory convergence towards steady state*" – refers to the cbe series with CVEC or CVCEC>0.1, which consists of 24 series. The multiple cycles have been built, including 2, 3, and even 5 primary cycles when possible. Nevertheless, the coefficient of variation did not drop under the 0.1 adopted ceiling. The data are consolidated in Appendix 11 (which is built identically as the previous one).

Based on the above-delimited typology, the global distribution of the examined cbe series is presented in Figure 6.

Figure 6

Dynamic Typology of Post-Sample Extrapolations
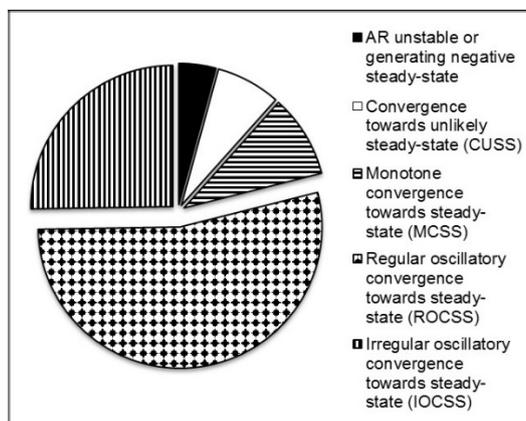
The preponderance of the oscillatory (particularly regular) convergence towards steady state should be a subject of future studies from both statistical and economic perspectives. The analysis of the differentiated lengths registered by the extrapolating cycles also represents a challenge.

3.2. Structural Approach

Structurally, the size of the public budget is interconnected with many other macroeconomic variables. One of these, possibly the most important, is economic growth.

This interconnection is the main reason that our attention will hereinafter be directed to the so-called BARS curve, which is linked with the seminal research of Barro (1990, 1991), Armey (1995), Rahn (1996), and Scully (1989, 1995, 1998). There is a large literature devoted to this relationship (some supplementary contributions are quoted in Dobrescu (2015a).

According to the BARS curve, which has been significantly corroborated by empirical studies, the dynamics of global output can be synthetically represented as a function (a parabola with a maximum) of the ratio of public budget expenditures to GDP (the cbe series in our paper).

We shall examine the simplest form of such a relationship:

$$IGDPc = f(cbe, cbe^2) \tag{6}$$

where: IGDPc represents the yearly index of the gross domestic product at constant prices.

This maximally compressed specification provides acceptable computational results in many cases. In others, however, the complete absence of non-budget influences proves too dangerous for regression robustness. Consequently, the effect of accumulation has occasionally been added to the factors cbe and cbe^2. As in other applications, this accumulation is represented here by the aICF parameter, defined as a two-term geometrical moving average:

$$aICF=(IGCFc*IGCFc(-1))^{0.5} \qquad (7)$$

Also involved in other specifications of the BARS curve, aICF considers both the current and lagged effects of investments. For 25 series, therefore, the relationship,

$$IGDPc=f(cbe, cbe^2, aICF) \qquad (8)$$

was used in estimations.

Another technical question concerning the BARS curve specification is collinearity, which is of concern not only for cbe and cbe^2 but also for aICF. Formula (8) has been preliminarily computed using OLS. The variance inflation factor reached alarmingly high levels in the overwhelming majority of cases (Supplementary material S10).

Consequently, instrumental variables were used, which is not at all a simple solution because of the diversity of the samples employed. The choice for such a role of lagged value for each explanatory variable has imposed itself as an almost uniquely accessible solution. However, the choice is conditioned by the lack of autocorrelation in residuals; see Angrist and Krueger (2001), Powell, SAS Institute Inc. (2010) and Wooldridge (2009). Computed in OLS using the same specification (three explanatories), the System Residual Portmanteau Tests for Autocorrelations (up to 6 lags) showed sufficiently high probabilities in favor of the "null hypothesis: no residual autocorrelations up to indicated lag". Therefore, the first lag for each exogenous variable has been adopted as an instrument.

The system of 95 equations, combining two- and three-factor specifications with the mentioned instruments, was solved with two-stage least squares and weighted two-stage least squares procedures. Under available data and the described specification, other techniques usable for systems with instrumental variables (seemingly unrelated regression (SUR), three-stage least squares (3SLS), generalized method of moments (GMM), and limited information maximum likelihood (LIML) failed.

The Supplementary material S11 presents in detail the results provided by the weighted two-stage least squares method. Clearly, in a more general notation, if the econometric coefficients are "a" for cbe and "b" for cbe^2, the maximum of IGDPc is reached when cbe=a/(-2*b). This value can be admitted as a possible structural attractor for the series of interest.

Estimated by the BARS curve, the attractors of the public budget size are shown in Appendix 12 for the entire available sample (all 95 countries).

Therefore, the structural approach provides estimations for all the countries when the group "AR unstable or generating negative steady state" is absent, which confirms the already-announced supposition that the inability of the autoregressive technique to reveal the steady state in some situations does not mean that a possible attractor cannot be found using other techniques. Obviously, the confidence level in the possibility of an attractor's presence becomes higher when more algorithms support that presence.

One should note that the BARS curve generates attractors closer to the sample mean than the AR process. A comparison will be made for the common sample of 91 countries. The relative attractor-mean deviation (rAM2, determined by formula (4) applied on ATT2) is distributed as shown in Appendix 8: 6 cases under 0.01, 33 between 0.01 and 0.05, 22 between 0.05 and 0.1, 28 between 0.1 and 0.75, and only two beyond this limit.

Figure 7 describes visually the relative attractor-mean deviation (in the module) for both autoregressive and structural attractor approaches.

Figure 7
The Relative Attractor-Mean Deviation (in Module) for Autoregressive (rAM1) and Structural (rAM2) Variants
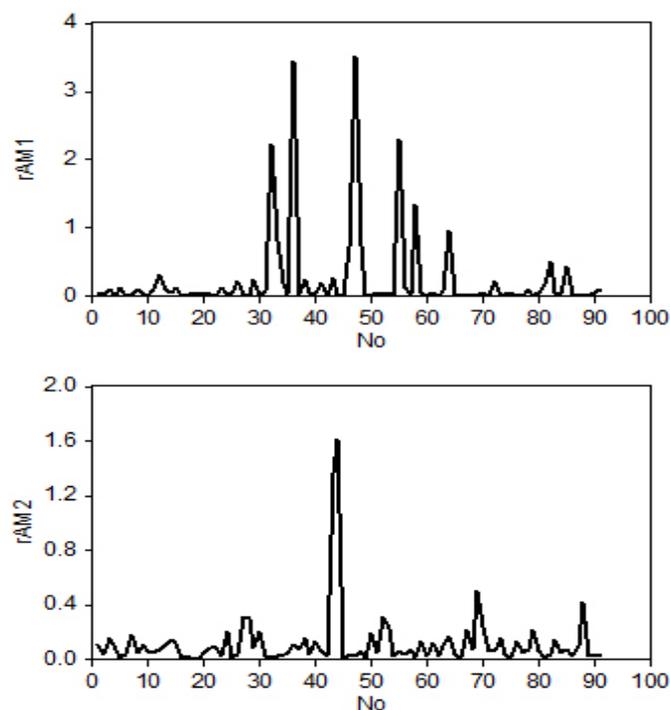
Evidently, in contrast with the VAR approach, the BARS curve does not offer any information regarding the trajectory of the cbe series towards the corresponding attractors. Whether such a difference might be useful in analyzing the dynamics of controversies surrounding the public budget size requires further analysis.

4. Conclusions

1. Due to the complex role exerted by the public budget in institutional governance – securing some services vital for all citizens (education, health care, justice and public order, national defense, environmental protection), the reallocation of resources through the taxes and subsidies, favoring stability and economic growth, infrastructure modernization, promoting the social solidarity, reduction of regional disparities - the dimension of government expenditures and revenues has established itself as a leading macroeconomic problem. Reflecting the transition from the minimal to developmental state, the so-named Wagner's Law considerably amplified the significance of this issue.

2. The debates have especially focused on the public budget size repercussions on the global output of the economy. Three viewpoints proved dominant.

The first outlined the direct and propagated positive effects of public expenditures, involving not only the Keynesian doctrinal framework (demand-side incentives), but also the supply-side perspective (human capital accumulation, research-development programs, infrastructure investment).

Secondly, the negative consequences of the state intervention in economic life, including through public finance policies, were also systematically revealed.

Finally, the opinions insisting on the ineffective (at best) role of government revenues and management for economic growth did not lack as well. This conceptual puzzle was supported by a lot of empirical case studies.

The paper attempted to answer the question: "Are the controversies surrounding the size of the public budget accidental from an economic point of view?"

3. With this aim, a significant statistical sample comprising 95 countries for the 1990-2012 period was subjected to a relatively large battery of tests; the variance ratio was also corroborated with stationarity procedures. Supplementary, autocorrelation, Ljung-Box Q statistics, the BDS test, coefficient d from the ARFIMA algorithm, and the Hurst exponent were employed to check whether the given data were free from temporal interdependence.

Some signs of random walk behavior in the series examined were identified, which *per se* might accredit the presumption that controversies around public budget size would not have economic roots as such, as it would instead by an expression of the volatile psychosocial and political circumstances.

4. Despite these indications, the dominant finding was the presence of serial correlation in most parts of the data. This correlation directed us to seek possible attractors as gravitational points for the academic or public opinion debates concerning the government expenditures' proportion in GDP.

As preliminary computational techniques, the paper applied a VAR algorithm to available series to identify the steady state by asymptotical value of a moving average auto-regressive process and a regression with instrumental variables for estimating the optimal point of the BARS curve. The results proved encouraging in most cases.

If such 'latent magnets' exist, than the oscillations of social perceptions regarding the public budget dimensions could be at least partially explained by the effective position of the given economy compared with the applicable attractors. Representing this dynamic in the simplest manner, that is, as a parabola with a maximum, the economy could be situated in the proximity of its optimal relative public budget size or, in contrast, at the left (on the increasing slope) or at the right (descending trajectory) of the curve. It seems plausible to admit that as a mass phenomenon, social perception – deliberately or implicitly – would exteriorize the tensions thus induced. Figure 8 sketches this hypothetical mechanism.

Figure 8

A Hypothetical Mechanism of Social Perception Regarding the Size of the Public Budget.
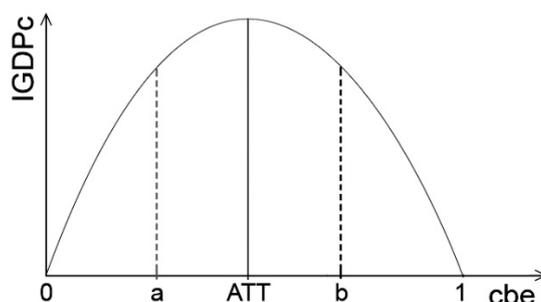
Note: 0-a: Growing public expenditure desirability; a-b; Accentuating public expenditure aversion; b-1: Extending agreement for public expenditure reduction

5. Our paper is only a small step in the direction of solving this problem. We have limited ourselves to an examination of the public budget-size attractor hypothesis in its essence and to the most accessible forms in terms of computation: the longest stable auto-regressive version and BARS curve, in both cases as points (steady state or extreme). In this matter, many investigating paths remain to be explored.

More algorithms should be tested. The quantitative analysis of more-realistic forms of possible attractors would also be rather challenging, such as the band of variable, macroeconomic anchors for the public budget size other than GDP, and the vectorial definition of this size. The technical problems also should be perfected with an extended psycho-sociological study that reveals the deep behavioral matrix of the phenomenon discussed herein.

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