3. EVALUATION OF THE DEGREE OF UNCERTAINTY IN THE TYPE-2 FUZZY LOGIC SYSTEM FOR FORECASTING STOCK INDEX

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Abstract

The paper deals with investment analysis based on a new fuzzy methodology. Specifically, the interval type-2 fuzzy logic model is created to support decision-making for investors, financial analysts and brokers. The model is demonstrated on the time series of the leading stock index S&P 500 of the US market. Type-2 fuzzy logic membership features are able to include additional uncertainty resulting from unclear, uncertain or inaccurate financial data that are selected as inputs to the model.The paper deals mainly with the evaluation and comparison of different degrees of uncertainty of the functions of the membership of input variables. Several model situations with different levels of inaccuracy are created. Based on the results of the comparison, it can be said that the type-2 fuzzy logic with dual membership functions is able to better describe data from financial time series.

Keywords: computational finance, fuzzy logic, type-1 fuzzy logic, T1FLS, type-2 fuzzy logic, T2FLS.

JEL Classification: C63, G11, G15, G17

1. Introduction

A part of all economies is the financial system, as it is the most important channel for allocating funds according to Billha et al. (2016). The fundamental challenge in investment management lies in the correct timing of investing funds in financial market instruments so that investors achieve the expected return while taking into account an acceptable level of risk and evaluate their savings. Basic financial theories are based on simplifying assumptions that can cause significant problems and lead to losses when applied in practice. Forecasting tools are often used to identify the right moment to enter the financial markets in order to determine future price developments. However, this is a very complicated task,

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especially due to the behavior of financial markets, which are highly unstable to chaotic and there is a high degree of uncertainty, as described by Rajab and Sharm (2019). For that reason, it is important to create an expert model that is able to understand and integrate this behavior. In addition to the typical behavior of the financial market, it is necessary to take into account a number of factors that influence the development of financial instruments (Chen et al., 2016). Individual investors thus create their own models and combine these factors into models based on subjective judgment. As noted by Chang et al. (2011) successful financial market forecasting requires an appropriate tool that is able to deal with price uncertainty and unstable market behavior.

Although there are many approaches to predicting the development of market prices or the volatility of stock markets, it cannot be said that there is a universal model. In recent years, research has been strongly concerned with the issue of the optimal predictive model for financial markets. Even the slightest improvement in prediction accuracy can bring investors a significantly higher appreciation of their funds. Standard statistical models are quite limited for financial markets mainly due to non-linear development. For that reason, models that are able to overcome the problem of price uncertainty, noise and unstable development are gaining popularity, since financial markets are known to be affected by random and deterministic factors. Fuzzy logic, neural networks and hybrid models are among the widely applied models that are able to overcome traditional statistical tools.

Fuzzy logic (FL) is able to integrate additional uncertainty and imprecision, as Janková et al. (2021) and work with natural language in a relatively simple way. In other words, it is capable of containing vaguely characterized expert knowledge. Lotfi A. Zadeh (1965), the main representative of fuzzy logic, called this method the principle of incompatibility. Vague information is extremely important in the real world, because every person makes decisions based on inaccurate, vague or incomplete information that he receives from the outside world. Fuzzy logic, thanks to its ability to integrate these inaccuracies or uncertainties, improves the accuracy of predictions, as it is able to bridge the gap between quantitative and qualitative data.

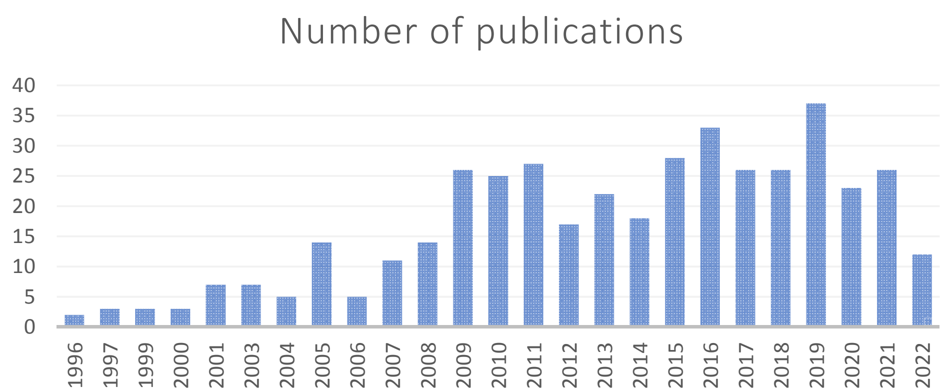
Several researches regarding the application of fuzzy logic to predict the development of the stock market can be found, see for example Wang and Wang (2015) and Rao et al. (2017). The motivation for researching this area lies in the fact that the mentioned research points to the improvement of the performance and accuracy of prediction when using fuzzy logic. In the recent application field, a higher level of fuzzy logic, which is able to contain additional uncertainty, is coming to the fore. It can be assumed that this type of fuzzy logic, referred to as type-2 fuzzy logic, is able to better describe the development of stock markets, which are characterized by highly volatile, chaotic and unstable behavior. However, marginal attention is paid to this issue in the context of economic sciences in the scientific community. For that reason, the subject of this paper is to introduce and apply type-2 fuzzy logic (T2FLS), which is still insufficiently explored in the economic or financial sphere. A fundamental contribution is the application of T2FLS, which, unlike T1FLS, is an almost unexplored area. In addition, several model situations that differ in the level of integrated uncertainty v. The performance of individual models across different levels of uncertainty is examined, which are subsequently compared.

The paper is structured as follows. Section 2 overview of the current state of knowledge of the application of fuzzy logic in stock markets. Section 3 introduction of fuzzy logic and detailed analysis of type-2 fuzzy logic. Section 4 presents experimental results and comparison of individual models. Section 5 summarizes and synthesizes the findings, benefits and limitations of the presented research.

2. Fuzzy Logic in Financial Market

Fuzzy logic has been widely used in the financial markets for decades, as declared by Wang and Wang (2015) ,who state that the art of prediction of fuzzy logic is widely recognized mainly due to its ability to capture nonlinear behavior. A total of 426 publications are selected from the Web of Science database based on keywords. The development of the number of publications in the field of fuzzy logic with application to financial markets is shown in Figure 1. The figure shows the growing trend of the application of fuzzy logic for predicting the development of the financial market.

Figure 1. Development of the Number of Publications T1FL



Rao et al. (2017) use fuzzy logic for investment portfolios to facilitate investment decisions. Similarly, Janková and Dostál (2019) deal with the use of fuzzy logic in the optimization of an investment portfolio as an effective tool in the upcoming digital era. Both studies state that fuzzy logic is a suitable tool to facilitate investment decisions. Other interesting conclusions can be found in the earlier work of Othman and Schneider (2010), where the authors add that fuzzy logic is a simple tool and very beneficial for investors. Dourra and Pepe (2002) highlight the strengths of FL stating that this kind of logic simulates human behavior in investment trading. To solve the complexity of the stock market, the authors recommend applying fuzzy logic.

Table 1 summarizes recent studies focused on predicting the development of the financial market. It can be noted that the application is mainly used to predict the development of a stock index or stock title, then to create a portfolio and a suitable investment strategy, and last but not least, for other purposes such as predicting or identifying a turbulent period in the financial markets. These are the most common application areas found in the last few years. According to research by Rao et al. (2017), fuzzy models can be used to generate investment decisions that are authoritative for investors minimizing risk for their investment portfolios in the long term.

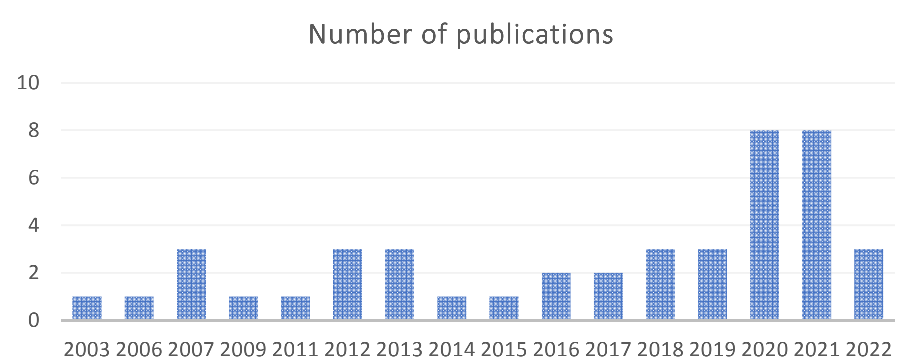
Table 1. Summary of previous T1FL on predicition of financial markets

Author/s	Model	Market	Stock/index	Output
Mohamed et al. (2021)	ANFIS	UAE	stocks	development prediction
Wang et al. (2020)	Fuzzy model	China	index	identification of turbulence
Chourmouziadis et al. (2021)	Fuzzy model	Greece	portfolio	optimal level of the investor's portfolio
Do and Trang (2020)	ANFIS	Vietnam	index	development prediction
Chandar (2019)	ANFIS	USA	stocks	development prediction
Fanita and Rustam (2018)	ANFIS	Indonesia	index	prediction and classification
Garcia et al. (2018)	ANFIS	Germany	index	development prediction
Atsalakis et al. (2016)	Fuzzy model	USA	stocks	identification of turbulence

In their work, Dourra and Pepe (2002) use fuzzy information technology for investment trading with the integration of technical indicators. The results show that the fuzzy model is able to solve the complexities of the stock market.

The research discussed above exclusively applies type-1 fuzzy logic (T1FL). In T1FL, exact degrees of membership are assigned in fuzzy sets, which is not entirely relevant for the area of financial markets. However, this exact degree of membership means that type-1 fuzzy sets are not able to handle problems involving uncertainty, such as noisy or non-stationary conditions. For that reason, a new type of fuzzy logic was created, namely type-2 fuzzy logic (T2FL), which is able to integrate the additional uncertainty related to the membership function. The fundamental importance of the application of type-2 fuzzy logic is the ability to better deal with chaotic time series compared to type-1 fuzzy logic, as noted by Karnik and Mendel (1999). Currently, there is only a limited amount of work dealing with the application of type-2 fuzzy logic. Similarly, Liu et al. (2012) and Khosravi et al. (2012) report that T2FLs deal better with uncertainties and improve short-term prediction. The development of the number of publications is shown in Figure 2.

Figure 2. Development of the Number of Publications T2FL



It is evident that the number of publications in this area is very low, which ilustrates the fact that this is a relatively new and applicationally unexplored topic in the economic domain.

Table 2 lists significant studies in the T2FLS application. It is obvious that, in addition to classical prediction, this higher type of fuzzy logic is also widely used to create decision-making models. Jiang et al. (2018) propose an IT2FLS for stock index forecasting based on a fuzzy time series and a fuzzy logical relationship map (FLRM). The application of IT2FLS in the context of portfolio theory and its rebalancing can be found in the study by Pai (2017), who additionally selects the model according to the investor's level of aspiration. A new approach for predicting stock indices through IT2FLS was presented in their research by Bhattacharya et al. (2016). According to the results of the research, the implemented expert model successfully competes and significantly outperforms the prediction algorithms used so far.

Table 2. Summary of Previous T2FL on Predicition of Financial Markets

Author/s	Model	Market	Stock/index	Output
Takahashi and Takahashi (2021)	IT2FLS	USA, Japan, Germany, UK	assets	decision-making model
Janková and Dostál (2021)	IT2FLS	USA	ETF	decision-making model
Jiang et al. (2018)	IT2FLS	Taiwan, USA	index	development prediction
Pai (2017)	IT2FLS	India	portfolio	rebalancing portfolio
Bhattacharya et al. (2016)	IT2FLS	USA	index	development prediction

Based on the literature review, the following outputs are synthesized: (1) A properly designed and balanced expert fuzzy model is able to increase the accuracy of time series prediction with regard to their interpretation. 2) The exact degree of membership of fuzzy type 1 sets means that they are not able to solve problems associated with uncertainty, such as stock index fluctuations. On the other hand, type-2 fuzzy sets are able to handle these problems integrated between membership functions. The presented research deals with the application of IT2FLS integrating type-2 fuzzy sets. IT2FLS is designed to predict the development of the stock index, while several models with different levels of uncertainty of the fuzzy membership function are created.

3. The Fuzzy Logic System

Fuzzy logic (FL), unlike classical logic, is able to integrate the uncertainty and ambiguity contained in the real data of real-world problems. The representative of fuzzy logic prof. Zadeh (1965) developed this approach because it better reflects human judgment and is able to work with linguistic expressions that cannot be easily quantified by numerical values. Zadeh coined the term "fuzzy" denoting vagueness, indeterminacy or ambiguity used to replicate human language. The power of fuzzy logic thus clearly lies in the ability to process linguistic information and make decisions similar to human judgment. In addition, fuzzy logic is able to provide a platform for modeling conditions and situations that are inherently imprecisely defined or uncertain to chaotic. Fuzzy sets are the backbone of all fuzzy logic and are immune to all kinds of uncertainties that are prevalent in the real world across various disciplines. Unlike classical sets, which are sharp and bivalent showing only two data - true or false, fuzzy sets represent extensions in the sense that they allow partial membership. Fuzzy sets are defined by a membership function that takes any value in the interval [0,1].

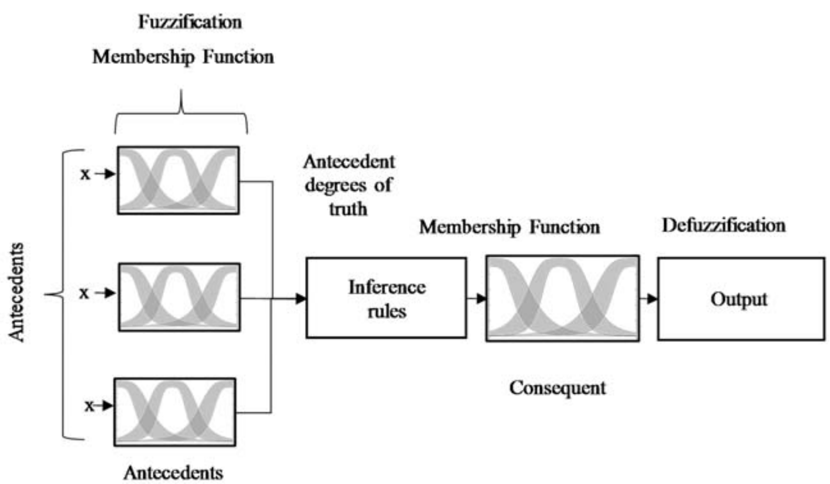
FLS has experienced two important stages of development since 1970: the classic type-1 FLS (T1) using the fuzzy set T1 and the advanced type-2 FLS (T2) using the fuzzy set T2. Although T1FLS has been comprehensively studied from both a theoretical and an application perspective, T2FLS has attracted more attention in the last decade due to its excellent performance in modeling the uncertainty we often encounter when modeling tasks in the real world.

Type-2 Fuzzy Logic System

The principle of operation of both types of fuzzy logic is very similar. The structure of T2FLS is shown in Figure 3. At the beginning, the measured real data are fed into the fuzzification block, where the transformation into linguistic variables takes place. Each linguistic variable is further divided into attributes. The number of attributes is usually between three and seven (Janková and Dostál, 2021).

The attributes of individual input variables in the fuzzy set are represented by a mathematical function. In the T2FLS expert system, there are three types of output variables: (1) if the obtained input data is perfect, the output is modeled as a sharp set; (2) if the input contains noise or stationary noise, the output is generated as a type-1 fuzzy set; (2) data with non-stationary noise is generated as a type-2 fuzzy set at the output.

Figure 3. Diagram of fuzzy processing in IT2FL



The mathematical notation of the possibility of a distribution function that represents T2FLS is according to Mendel et al. (2006) and Sang et al. (2019) as follows:

$$\tilde{A} = \int_{x\epsilon X} \int_{u\epsilon J_x} \frac{\mu_{\tilde{A}}(x,\mu)}{x,u} = \int_{x\epsilon X} \left[\int_{u\epsilon J_x} \mu_{\tilde{A}}(x,\mu)/(x,u)\right]/x, \tag{1}$$

where x is the primary variable $J_x\epsilon[0,1]$ is the primary membership of x, u is the second variable, and $\int_{u\epsilon J_x} \mu_{\tilde{A}}(x,\mu)/(x,u)$ je secondary possibility distribution at x.

The condition of normality is a necessary requirement for the secondary possibility distribution, because only then are the elements fully distributed as shown below:

$$\tilde{A} = \int_{x \epsilon X} \int_{u \epsilon J_x} \frac{1}{x,u} = \int_{x \epsilon X} \left[\int_{u \epsilon J_x} 1/(x,u) \right] /x, \tag{2}$$

where are lower possibility distribution $\underline{\mu(x)}$ and are upper possibility distribution $\overline{\mu}(x)$ and type-1 possibility distribution, and according to Mendel et al. (2006) the footprint of uncertainty $\tilde{X}(FOU(\widetilde{X}))$:

$$FOU(\tilde{X}) = \cup_{x \in X} J_s = \left\{ (x,y) : J_X = \left[\overline{\mu}(x), \underline{\mu}(x) \right] \right\}, \tag{3}$$

IT2FLS $\tilde{X} = \left[\bar{\mu}(x), \underline{\mu}(x) \right] = ((a,b,d;h^U),(e,b,f;h^L))$, where $\underline{\overline{\mu}(x)}$ and $\underline{\mu}(x)$ are T1FLS, a,b,d,e,f are reference points of the IT2FLS, h^U indicates the possible value of the element a,b,d in the upper possibility function, h^L indicates the possibility value of the element e,b,f in lower possibility function, $h^U \epsilon [0,1]$, and $h^L \epsilon [0,1]$. The lower and upper possibility distribution is defined as:

$$\overline{\mu}^U(x) = \begin{cases} \frac{\mu^U(x-a)}{b-a}, & a \le x \le b \\ \frac{\mu^U(d-x)}{d-b}, & b \le x \le d \\ 0, & otherwise \end{cases}, \tag{4}$$

$$\underline{\mu}^L(x) = \begin{cases} \frac{\mu^L(x-e)}{b-e}, & e \le x \le b \\ \frac{\mu^L(f-x)}{f-b}, & b \le x \le f \\ 0, & otherwise \end{cases}, \tag{5}$$

For IT2FLS $\tilde{X} = ((a,b,d;h^U),(e,b,f;h^L))$, whose possibility uncertainty mean value can be calculated by to the following notation by Sang and Liu (2016):

$$M(\tilde{X}) = \frac{\widetilde{M(X^U) + M(X^L)}}{2}, \tag{6}$$

where upper MF $M(\widetilde{X^U})$ and lower MF $M(\widetilde{X^L})$ mean possibility uncertainty:

$$M(\widetilde{X^U}) = 1/2 \int_0^{h^U} \left(\underline{X^U}(\alpha) + \overline{X^U}(\alpha) + 2b \right) f(\alpha) d\alpha, \tag{7}$$

$$M(\widetilde{X^L}) = 1/2 \int_0^{h^L} \left(\underline{X^L}(\alpha) + \overline{X^L}(\alpha) + 2b \right) f(\alpha) d\alpha, \tag{8}$$

$f(r)$ is a function satisfying $f(0) = 0$, $f(1) = 1$ and $\int_0^{h^U} f(\alpha) d\alpha = 1/2$.

According to Sang and Liu (2016), the variation of possibility uncertainty IT2FLS is calculated by the formula:

$$VC(\widetilde{X}) = \begin{cases} \frac{D(\widetilde{X})}{M(\tilde{X})}, & if\ M(\tilde{X}) \neq 0 \\ \frac{D(\tilde{X})}{\epsilon}, & if\ M(\tilde{X}) = 0 \end{cases}, \tag{9}$$

where ϵ is a very low value approximating the variation values $M(\tilde{X}), D(\tilde{X})$. $D(\tilde{X})$ is defined as:

$$D(X) = \sqrt{\widetilde{D(\widetilde{X^U})\ D(\widetilde{X^L})}},$$

$$D(\widetilde{X^U}) = 1/4 \int_0^{h^U} \left(\overline{X^U}(\alpha) - \underline{X^U}(\alpha)\right)^2 f(\alpha) d\alpha, \qquad (10)$$

$$D(\widetilde{X^L}) = 1/4 \int_0^{h^L} \left(\overline{X^L}(\alpha) - \underline{X^L}(\alpha)\right)^2 f(\alpha) d\alpha,$$

where $f(\alpha)$ is a function satisfying $f(0) = 0, f(1) = 1$ and $\int_0^{h^U} f(\alpha) d\alpha = 1/2$.

4. Experimental Results and Analysis

The following section introduces the application of type-2 fuzzy logic of various levels of uncertainty to the US stock market. Specifically, the main stock index Standard & Poor's 500 is selected on a daily basis for the period 2015 to 2019. The created model of a higher degree of fuzzy logic is created to support decision-making for investors, brokers or the general investor public. Type-2 fuzzy logic is recommended if the dataset sample is inaccurate, uncertain, or contains missing data. Due to the fact that financial data are showing this nature, it is obvious to use just this new fuzzy methodology.

Data Sample

Standard & Poor's 500 (S&P 500) is a stock index containing the 500 largest publicly traded corporations in the United States. The index includes so-called large cap companies, these are companies that have a high market capitalization. It must not fall below 6 billion US dollars and the volume of publicly traded shares must not be less than 250,000 thousand per month. Not all companies that are included in the index have an equal position here. Each company has a different weight in the index. Different indices use different calculations to consider the importance of a particular company. In the S&P 500 index, shares are weighted according to market capitalization, which means that not only the price of individual shares decides, but also their number. The companies with the largest market capitalization therefore have the greatest weight in the S&P 500 index.

Figure 4. Development of the S&P 500 Stock Index

This index is one of the most important stock indices in the world. The S&P 500, like all major indices, uses the Global Industry Classification Standard (GICS) to divide companies into industries such as energy, healthcare, finance, information technology and retail. The general investor public considers it the best representation of the American stock market and an indicator not only of the American economy.

Tabel 3 summarizes the basic statistical data of the selected American stock index. The average price of the S&P 500 index for the period under review was 2452.64. The maximum value to which the index climbed is 3240.02, and on the contrary, it fell to the level of 1829.08. To calculate the volatility of the American stock market benchmark, the standard deviation is chosen, which shows a value of 357.31. According to the skewness value, it can be noted that the index shows a positive asymmetric probability distribution. According to the Kurtois values, it can be summarized that the index does not show a Gaussian normal distribution.

Table 3. Summary Statistics of S&P 500 Index

Statistics	Index
Mean	2452.64
Standard deviation	357.31
Minimum	1829.08
Maximum	3240.02
Skewness	0.18
Kurtois	-1.29

The closing prices of the selected stock index are chosen on a daily basis for the monitored period 2015 to 2019 are used. In other words, the medium-term horizon is tested, i.e., a 5-year investment period. It is necessary to preprocess the time series according to the following standardization procedures according to Li and Xiong (2005) and Zhai et al. (2010). This pre-processing step is performed due to large differences in index prices over the observed period, which could cause degradation of the model's performance.

$$y_t = \frac{x_t - m}{M - m} \tag{11}$$

where x_t is the closing daily price of the stock index at time $t, M = max\{x_t\}$ and $m = min\{x_t\}$. The variables entering the model can be written as:

$$(y_{t-3}, y_{t-2}, y_{t-1}, y_t) \tag{12}$$

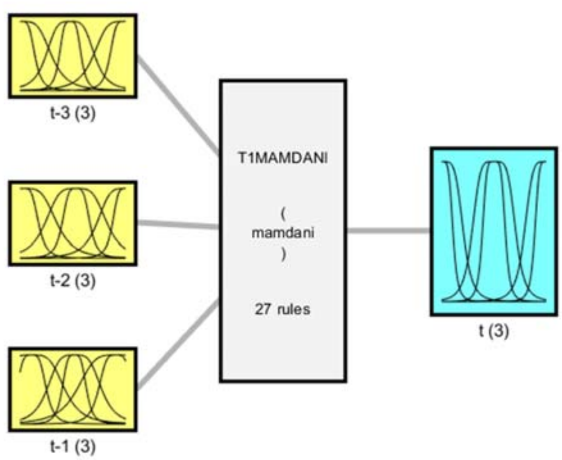
Table 4. Sample dataset of S&P 500 Index

Date	Input 1 (t−3)	Input 2 (t−2)	Input 3 (t−1)	Output (t)
22.02.2016	0.08251	0.06534	0.07139	0.08691
23.02.2016	0.06534	0.07139	0.08691	0.08432
24.02.2016	0.07139	0.08691	0.08432	0.07311
25.02.2016	0.08691	0.08432	0.07311	0.10579
26.02.2016	0.08432	0.07311	0.10579	0.11154

Experimental Results

An expert model interval type-2 fuzzy inference system (IT2FIS) of Mamdani type is created. The model contains three input variables and one output variable that predicts the normalized price of the stock index. Due to the nature of stock market data, Bell membership functions are selected to best suit their nature. In addition, based on a preliminary survey, the bell function showed the lowest error rate compared to other membership functions. The input and output variables of the model are divided into three attributes (LOW, MEDIUM, HIGH). The actual structure of the T2FIS model is depicted in Figure 5.

Figure 5. T2FIS Architecture of the S&P 500 Index



Rules in the form of IF-THEN are generated by experts and there are a total of 27 of them in the proposed Mamdani fuzzy model. Three sets of type-2 fuzzy sets are constructed to linguistically evaluate selected fundamental indicators (low, medium and high value). The value of the LowerScale parameter defines the maximum value of the lower membership function and LowerLag indicates the size of the integrating additional uncertainty between the doubled membership functions. The LowerScale parameter is a scale with a positive value at most equal to 1. The LowerLag parameter expresses the delay, or the point at which the value of the lower membership function differs from 0 based on the values of the higher membership function. For example, if LowerLag is set to 0.1, it means that once the upper membership function reaches 0.1, the lower membership function becomes positive. In this study, several sample cases of models that differ from each other in these parameters are evaluated.

Comparison of Results

Based on the procedure described above, several T2FLS models are created differing from each other by the level of uncertainty integrated between the membership functions.

A total of 6 experimental models are created according to different degrees of uncertainty. Table 5 contains in columns the shapes of the membership functions of all three inputs to the fuzzy model, including the included additional uncertainty, which differs for individual model situations. Model (1) does not contain any additional uncertainty, it is a T1FLS model. Model (2) contains fuzzy sets with lower scale 0.9 and lower lag 0.1. Model (3) integrates 20% additional uncertainty, which is graphically represented by the gray flat between the upper and lower membership functions. Model (4) integrates 30% uncertainty and model (5) respectively, (6) 40% uncertainty respectively, 50% uncertainty.

A training data set is used for training the expert model. Subsequently, the accuracy of the trained model is tested using the test data set. The ratio of the training and testing data set is chosen in the ratio of 80:20, while the most recent data is used for testing the trained fuzzy model. The obtained results are then compared according to the error rate indicators. The basic indicator that is calculated is the RMSE, in the calculation of which the original data y_t and the data obtained from the output of the model $\dot{y}_t$ are compared. Mean Absolute Percentage Error (MAPE), Mean Absolute Error (MAE), Relative Root Mean Squared Error (RMSE), Mean Squared Error (MSE). The mathematical formulas for calculating these indicators are:

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(y_t - \dot{y}_t)^2} \tag{13}$$

$$MAE = \frac{1}{n}\sum_{t=1}^{n}|y_t - \dot{y}_t| \tag{14}$$

$$MSE = \frac{1}{n}\sum_{t=1}^{n}(y_t - \dot{y}_t)^2 \tag{15}$$

Output values indicating the normalized price of the index are recorded in Tabel 6. These predicted values will subsequently be used to calculate error indicators and determine the performance of individual T2FLS models. Table 6 contains the output values predicting the development of the stock index for individual examined model situations differing in the level of uncertainty.

Table 5. Different Levels of Uncertainty of Member Functions

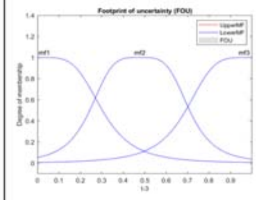
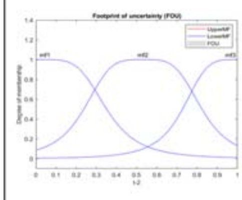
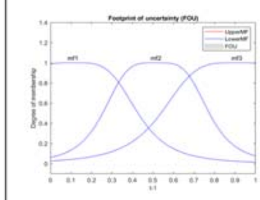
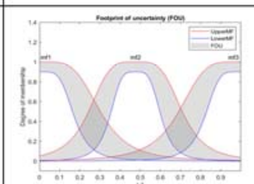
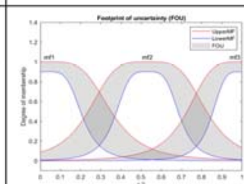
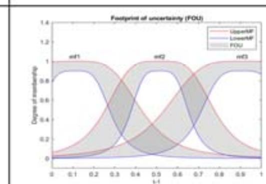
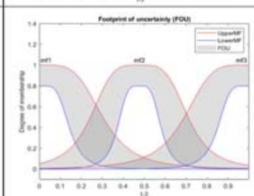
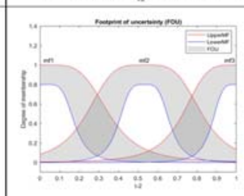
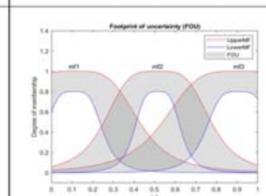
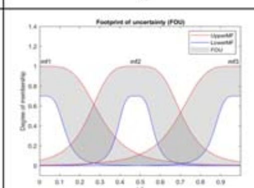
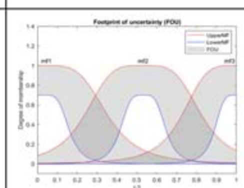
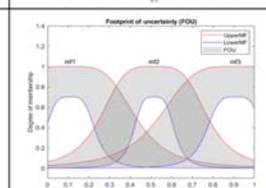
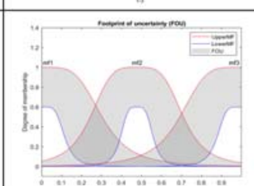
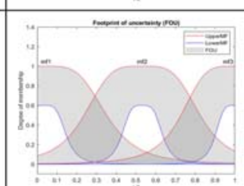
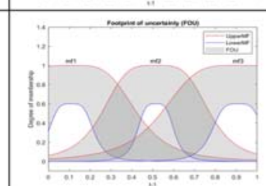
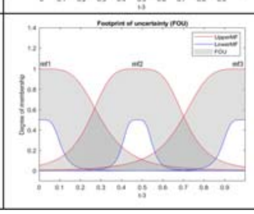
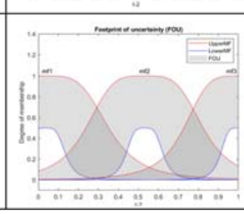
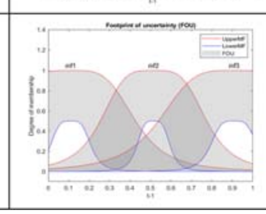
Model	Input 1	Input 2	Input 3
(1) LowerScale 1 LowerLag 0			
(2) LowerScale 0.9 LowerLag 0.1			
(3) LowerScale 0.8 LowerLag 0.2			
(4) LowerScale 0.7 LowerLag 0.3			
(5) LowerScale 0.6 LowerLag 0.4			
(6) LowerScale 0.5 LowerLag 0.5			

Table 6. Outputs from Individual Models According to Uncertainty Levels

Model (1)	Model (2)	Model (3)	Model (4)	Model (5)	Model (6)
0.3033	0.2715	0.2913	0.3122	0.3321	0.3511
0.2822	0.2405	0.2563	0.2769	0.2983	0.3202
0.3154	0.2906	0.3113	0.3312	0.3493	0.3664
0.3348	0.3063	0.3229	0.3402	0.3563	0.3719
0.3509	0.3381	0.3547	0.3696	0.3833	0.3974
0.3990	0.3869	0.3908	0.3974	0.4068	0.4204
0.4248	0.4288	0.4287	0.4297	0.4334	0.4402
0.4611	0.4707	0.4720	0.4715	0.4705	0.4694
0.5728	0.5603	0.5595	0.5610	0.5619	0.5610
0.5434	0.5408	0.5420	0.5396	0.5350	0.5300
0.6016	0.6059	0.6016	0.5955	0.5862	0.5735
0.6322	0.6385	0.6289	0.6197	0.6105	0.6002
0.5876	0.5907	0.5898	0.5861	0.5792	0.5685
0.5997	0.6057	0.6047	0.6014	0.5958	0.5865
0.6173	0.6239	0.6816	0.6123	0.6048	0.5947
0.7015	0.7327	0.7150	0.715	0.6287	0.6593

Tabel 7 shows the results of the error and performance indicators of the T2FLS model with different levels of uncertainty of membership functions on the examined time series of the S&P 500 stock index. For the MSE and MAE indicators, model 2 shows the lowest values, which indicates the lowest error rate. The error value for that model is 0.067, or 0.075. The worst results, i.e., the highest prediction error, can be found in models 5 and 6. The RMSE indicator is also chosen to compare the quality of the models. The RMSE is also chosen to compare the error rates of the models. Model 2 has a value of 0.0866, which indicates a lower error rate and thus a more accurate calculation compared to model 1 (0.1019), model 3 (0.950), model 4 (0.1068), model 5 (0.1245) and model 6 (0.1323).

Table 7. Comparison of Levels of Uncertainty

Model		RMSE	MSE	MAE
Model (1)	LowerScale 1 LowerLag 0	0.1019	0.0104	0.0889
Model (2)	LowerScale 0.9 LowerLag 0.1	0.0866	0.0075	0.0067
Model (3)	LowerScale 0.8 LowerLag 0.2	0.0950	0.0090	0.0823
Model (4)	LowerScale 0.7 LowerLag 0.3	0.1068	0.0114	0.0939
Model (5)	LowerScale 0.6 LowerLag 0.4	0.1245	0.0155	0.1074
Model (6)	LowerScale 0.5 LowerLag 0.5	0.1323	0.0175	0.1155

From this point of view, based on the error and performance indicators, it can be stated that the model with the lower scale 0.9 and lower lag 0.1 achieved the best results. In other words, it can be stated that the T2FLS model, which allows to include a certain degree of uncertainty using dual membership functions, provides better results compared to the classical T2FLS model, where it is not possible to record this higher degree of uncertainty. Especially for data files originating from financial markets. In further research, it would be useful to examine more closely the degree of uncertainty between models 1 and 3 that achieved the best results. It would not be interesting to examine in more detail the degree of uncertainty doubting between lower scale 1 to 0.8 and lower lag between 0 and 0.2.

Table 8. Comparison with Other Standard Models

Model	RMSE	MSE	MAE
Linear Regression	0.1081	0.0117	0.4608
Decision Tree	0.2429	0.0590	0.2220
Support Vector Machine	0.1269	0.0161	0.0995
Neural Network	0.0848	0.0075	0.0567

Table 8 shows the error rate of standard models used to predict the development of the stock market. Statistical regression analysis, decision trees, support vector machine and neural networks are chosen for comparison. It is clear from the table that decision trees show the highest error rate according to RMSE 0.2429, followed by SVM 0.1269 and linear regression 0.1081. A relatively low error rate can be observed with neural networks. From the point of view of MSE, the best fuzzy model shows the same value as neural networks 0.0075. The MAE of the best fuzzy model is significantly better than that of neural networks. It can be stated that the created T2FLS models with uncertainty integration can compete with standard models used for stock market prediction. It can also be said that with appropriate integration of uncertainty, a significantly lower error rate can be achieved rather than with models such as linear regression, SVM, decision trees or even neural networks.

Improving the accuracy of the prediction of the development of the stock index, thanks to the use of the IT2FL expert system, the model can be deployed in real time to predict the trend in the stock market, thereby increasing the profitability of investors. The improvement in prediction accuracy is based on the idea of a model based on human judgment and the integration of uncertainty that results from the highly volatile and chaotic behavior of the stock market. The expert model can serve as decision support and provide relevant recommendations for investors, so they can make sophisticated decisions about the short-term strategy of buying or selling shares of the selected stock index. This makes the entire investment process more complex.

5. Conclusion

The paper deals with the application of a higher degree of fuzzy logic in investment analysis. Specifically, type-2 fuzzy logic is applied to the leading S&P 500 stock index. A decision-making model is created to serve as a support for investing in the US stock market for financial analysts, brokers and professionals. The expert model based on type-2 fuzzy logic contains three input variables with the help of which the output value is predicted. A higher level of fuzzy logic is chosen due to the ability to integrate the additional uncertainty that results from the chaotic behavior of the stock market. The paper applies several different T2FLS models that differ in the level of uncertainty between the membership functions.

The presented paper presents several model situations of a higher level of fuzzy logic. T2FLS models are able to integrate the additional uncertainty that results from the chaotic, volatile and unstable environment characteristic of stock markets. This expert model is able to model this behavior by integrating additional uncertainty. The novelty of this paper is mainly in the presentation of the T2FLS model in the financial sphere. T2FLS is primarily widely applied to technical fields, however, the nature of fuzzy logic has excellent predictive capabilities in stock markets, as already stated in previous studies applying T1FLS. The possibility of applying T2FLS compared to T1FLS demonstrated a better ability to adapt to market conditions and higher prediction accuracy mainly due to the ability of T2FLS to integrate additional uncertainty.

However, it is also necessary to draw attention to the limits of the present research. The presented model integrates only one shape of the T2FLS membership function. It would be appropriate to further compare the above with other forms of the membership function. Tuning the model also concerns the number of attributes, setting rules, etc. Furthermore, it would be appropriate to implement the created fuzzy model on other stock markets and monitor the performance of the model in order to provide generalized outputs. The mentioned model is also focused exclusively on short-term prediction of the development of the stock market. Long-term development can be considered one of the future directions of research.

Acknowledgements

This paper was supported by the project No. FP-S-22-7977 'Modeling and optimization of processes in the corporate sphere' and No. FP-J-20-6246 'The Use of Artificial Intelligence in Business IV' from the Internal Grant Agency at Brno University of Technology.

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